PHYS 2170: General Physics 3 for Majors **Fall 2021**

Exam 1

- This exam is open book and open notes. You are not allowed to use any electronic devices besides a calculator, nor allowed to communicate at all about the questions on the exam with any other students for 24 hours after the exam ends.
- You have 2 hours to complete this exam (unless Disability Services has authorized a request for extended time: 50% extra time is 3 hours, 100% extra time is 4 hours). Good luck!

20 **Problem 1:** Two particles (1 and 2) of mass m approach each other, traveling at equal and opposite velocities: $\mathbf{v}_1 = +v\hat{\mathbf{x}}$ and $\mathbf{v}_2 = -v\hat{\mathbf{x}}$. They collide and form a single particle of mass M.

1.1. Explain why the particle of mass M will be at rest.

1.2. In Newtonian mechanics, we would say that $M = 2m$. In relativistic mechanics, what is M? Express your answer in terms of m, v and/or c.

1.3. If we do this experiment in real life at home, say with two balls of clay or Silly Putty, would it be easy to detect the failure of Newtonian mechanics?

Problem 2: A pair of particle accelerators is placed at $x = -D$ and $x = +D$; each is at rest in frame S. As viewed in S, each particle accelerator is capable of imparting a particle of mass 3 MeV/c^2 a kinetic energy of 2 MeV. At time $t = 0$, each accelerator shoots a particle towards the other. Let events A_- and A_+ denote the events corresponding to the left and right accelerators shooting particles towards each other, respectively. In frame S, the coordinates of A_- are $(ct, x) = (0, -D)$, while A_+ is $(ct, x) = (0, D)$.

15 **2A:** Let's begin by understanding the motion of particles in frame S.

2A.1. What is the total (rest + kinetic) energy of each particle after it is accelerated in frame S?

2A.2. What is the momentum of each particle in frame S?

2A.3. In terms of D and c, write the coordinates (ct, x) of the event C where the particles collide together.[1]

20 **2B:** Now, let us view this process from the perspective of frame S', in which the right-moving particle is at rest.

2B.1. In frame S', find coordinates for events A_-, A_+ and C.

2B.2. Is the left-moving or right-moving particle launched first (i.e. at an earlier coordinate time) in frame S'?

2B.3. What is the total distance traveled by the left-moving particle in frame S'?

2B.4. Must your answer to 2B.3 be related to the standard length contraction formula in an obvious way? Why or why not?

[1] *Hint:* Give a simple argument why the collision has to occur at $x = 0$.

20 **Problem 3:** In frame S, Planet A lies at coordinate $x = 0$, and planet B lies at coordinate $x = 4$ light years. At time $t = 0$, aliens begin to attack planet A, which sends out a distress call to Planet B.

3.1. What is the first coordinate time t_0, in frame S, at which B can learn of the attack on A? Why?

3.2. B immediately sends out a military ship to help planet A, which travels at a velocity $v = 4c/5$. As viewed in frame S, how long does it take for the ship to reach A?

3.3. The military ship has food reserves for the crew that last for 4 years. Will the crew starve or not during the journey to A? Explain your answer.

20 **Problem 4:** A photon of energy E collides with a particle of mass m, which is at rest; assume $E \gg mc^2$.

4.1. An amateur scientist thinks that it is possible to create a massive particle of mass $M \approx E/c^2$ after the collision. Explain why this is not, in fact, possible.

4.2. If a single massive particle of mass M is created during this collision, find its actual mass M.

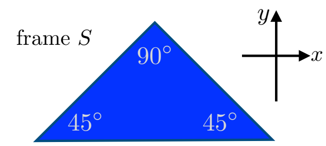

Figure 1: A 45-45-90 triangle at rest in frame S.

15 **Problem 5:** Consider an object with the shape of the "45-45-90" right triangle in Figure 1, at rest in reference frame S. In a reference frame S' moving at velocity $\mathbf{v}$ relative to S, the triangle appears to be an equilateral triangle. Find one possible value of $\mathbf{v}$.[2]

15 **Problem 6 (Relativistic mirror):** Consider a beam of light, incident from the right, which bounces off of a mirror which moves to the right at relativistic velocity $v = c\beta$. The goal of this problem is to relate the incident angle of light θ_i, to the reflected angle θ_r, as shown in Figure 2. The only things we know are Einstein's theory of relativity, along with the fact that when $\beta = 0$, $\theta_\mathrm{i} = \theta_\mathrm{r}$. Happily, this information is sufficient to solve the problem!

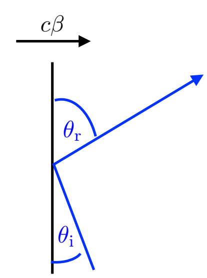

Figure 2: Light bouncing off a mirror moving at velocity $c\beta$.

6.1. Justify the following statement (which will prove useful in this problem): if $(ct_{1,2}, x_{1,2}, y_{1,2})$ denote the spacetime points of events 1 and 2, which lie along the trajectory of a photon that has not been deflected (e.g. by the mirror), then for some constants a and ϕ,

$$(ct_1 - ct_2, x_1 - x_2, y_1 - y_2) = (a, a\cos\phi, a\sin\phi). \qquad (1)$$

You may assume that $z = 0$ and do not need to keep track of this fourth coordinate.

6.2. Describe how photons bounce off of the mirror in its own rest frame by picking three cleverly chosen events that describe the trajectory of the photon. Then, Lorentz transform into a frame where the mirror is moving at the desired velocity, and show that

$$\sin\theta_\mathrm{r} = \frac{(1+\beta^2)\sin\theta_\mathrm{i} + 2\beta}{1 + \beta^2 + 2\beta\sin\theta_\mathrm{i}}. \qquad (2)$$

[2] *Hint:* What are the ratios of the base and height of the triangle in S, and in S'?