**PHYS 2170: General Physics 3 for Majors** **Fall 2021**

# Exam 2

- This exam is open book and open notes. You are not allowed to use any electronic devices besides a calculator, nor allowed to communicate at all about the questions on the exam with any other students for 24 hours after the exam ends.
- You have 2 hours to complete this exam (unless Disability Services has authorized a request for extended time: 50% extra time is 3 hours, 100% extra time is 4 hours). Good luck!

20 **Problem 1 (Acoustic guitar):** A guitar string of mass $m$ (uniformly distributed) and length $L$ is held fixed at both ends, and was wound to be under a tension of strength $T$.

1.1. What are the possible wavelengths of standing waves on the string?

1.2. What are the corresponding frequencies?

1.3. In a cold room, the speed of sound in air changes. Do you think the guitar string needs to be tightened (or loosened) in order to play notes at the same frequency as in a normal room?

**Problem 2:** Coherent light is incident normally on a diffraction grating of spacing $a$. If the grating is held in air (index of refraction $n \approx 1$) then one observes that when light is normally incident on the grating, the angle between the brightest spot(s) of light is $\theta \ll 1$ (measured in radians).

10 **2A:** Argue that the wavelength of light incident on the screen is $\lambda \approx a\theta$.

10 **2B:** Now, the grating is held up against a material whose index of refraction is $n = 2$.

2B.1. What is the wavelength of light in the new material? Why?

2B.2. What is the angle $\theta'$ between the two spots, as observed in the material?

15 **Problem 3:** Three coherent sources of light (X, Y, Z) are at $y = 0$ and $y = \pm a$, as shown in Figure 1. They each emit light at wavelength $\lambda$. Suppose that X emits light whose *amplitude* is twice as large as Y and Z. Let $\phi$ denote the *relative* phase between the source X, and Y/Z (which we assume are in phase). Let $I$ denote the intensity of light observed at point P at $x = b$. Assume $b \gg a, \lambda$.

3.1. Find a formula for $I$ as a function of $\phi$. Do not worry about any constant prefactors – focus only on the $\phi$-dependence in the answer. Your answer should take the form $I \sim \cos^2(\phi - \phi_0)$ for some constant $\phi_0$ (which you should determine).

3.2. For what $\phi$ is there constructive interference?

3.3. For what $\phi$ is there destructive interference?

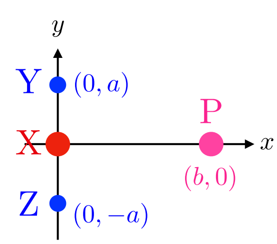


**Figure 1:** The points X, Y, Z and P.

**Problem 4:** An object is traveling towards a wall at velocity $v_0$. It emits a sound wave of frequency $f_0$ (in its own frame) towards the wall. The speed of sound in air is $v$. The wall and air are at rest in the same reference frame. Assume $v_0 < v \ll c$.

10 **4A:** What is the frequency $f_1$ of sound as measured by an observer standing at rest next to the wall?

10 **4B:** What is the frequency $f_2$ of the sound *reflected* off the wall, as measured by the object?

**Problem 5 (Optical phonons):** Phonons are the vibrations of a crystalline solid: the usual sound waves we discussed in class are only one kind of phonon. Another kind are called **optical phonons** (for awkward historical reasons); we can very crudely model them as follows. Let $u(x,t)$ denote the displacement of a block of solid. The dispersive wave equation describing the dynamics of $u$ is given by

$$\frac{\partial^2 u}{\partial t^2} = v^2 \frac{\partial^2 u}{\partial x^2} - \omega_0^2 u. \tag{1}$$

Here $v$ is a velocity scale and $\omega_0$ is a fixed angular frequency scale.

10 **5A:** Plug in the ansatz $u = \mathrm{e}^{\mathrm{i}kx - \mathrm{i}\omega t}$, and deduce the dispersion relation $\omega(k)$ of the optical phonon.

10 **5B:** What are the consequences of this dispersion relation?

5B.1. Calculate the phase velocity as a function of wave number $k$.

5B.2. Calculate the group velocity as a function of $k$.

5B.3. If we want to send information as fast as possible in this model, should we send it using a wave packet with a large or short overall length? Why?

15 **5C:** Now suppose that we consider these optical phonons in the finite domain $0 \le x \le L$. We fix the boundaries such that $u(0,t) = u(L,t) = 0$.

5C.1. Make the ansatz $u(x,t) = U(x)\mathrm{e}^{-\mathrm{i}\omega t}$. Plug in to (1); deduce a differential equation for $U(x)$.

5C.2. Solve this differential equation subject to the appropriate boundary conditions, and thus deduce the normal mode (angular) frequencies $\omega$.

5C.3. What are the wavelengths of the normal modes? Does the answer make sense?

10 **Problem 6 (Collision with a solid object):** Think of a baseball bat striking a baseball; after the collision, the baseball will begin to move at a finite velocity $v_*$. But microscopically, how does this happen? In this problem, we consider a model where the "baseball" is a solid 1d rod of length $L$. Suppose that *for sufficiently early times* $0 < t < (L-b)/v$, the displacement of the section of rod originally at position $0 \le x \le L$ is

$$u(x,t) = \begin{cases} \delta & 0 < x < vt \\ \delta\left(1 - \dfrac{x - vt}{b}\right) & vt < x < vt + b \\ 0 & vt + b < x \le L \end{cases}. \tag{2}$$

Assume that the boundary conditions on the rod are free: $\partial u/\partial x = 0$ at $x = 0$ and $x = L$. Assume that the displacement $u(x,t)$ obeys the usual (non-dispersive) wave equation for sound, with sound speed $v$. Here $\delta > 0$ is a small constant.

6.1. Describe, as quantitatively as you can, $u(x,t)$ for all times $t > 0$. It is not necessary to write down the explicit functional form of $u(x,t)$, but your answer should clearly convey understanding of what is happening.

6.2. Deduce an exact formula for $v_*$ (in this toy model).