PHYS 2170: General Physics 3 for Majors **Fall 2021**

Homework 1

Due: September 2 at 11:59 PM. Submit on Canvas.

30 **Problem 1 (Atomic clocks):** One of the specialties of the physics groups here in Boulder is atomic physics – in particular, the development of extremely precise atomic clocks. You can heuristically think of an atomic clock as a single atom which "resonates" (due to quantum mechanics) when driven at an extremely precise frequency f_0: experimentally, the resonance is only seen at frequencies f obeying

$$(1-\delta)f_0 < f < (1+\delta)f_0 \tag{1}$$

where the parameter $\delta \approx 10^{-18}$. Can we use this atomic clock to observe time dilation?

One experiment we could do is ask whether a "clock" attached to a moving atom would tick at a different frequency than a stationary one. Because the atomic clock can naturally "tick" at an extremely precise frequency f_0, even small effects will be measurable.[1] Suppose that one atom is at rest in the experimentalist's frame of reference and one atom is moving relative to the experimentalist at velocity v.

1.1. If a stationary atomic clock in the lab frame ticks every $T = 1/f_0$ seconds, what is the length of time T' in between ticks of the moving clock, if carefully measured by the experimentalist?

1.2. If we let $v = c\beta$, and note $\beta \ll 1$, Taylor expand your answer to show that

$$\frac{|T'-T|}{T} \approx \frac{\beta^2}{2}. \tag{2}$$

1.3. Due to (1), the experimentalist cannot exactly measure the atomic clock's ticking time: the measured time T_m is only guaranteed to obey

$$\frac{|T_\mathrm{m}-T'|}{T'} < \delta. \tag{3}$$

of the actual time T'. Conclude that we can only detect time dilation if

$$2\delta < \beta^2. \tag{4}$$

1.4. In the lab, we might be able to move an atomic clock safely at a speed of around 1 m/s. Show that even though this speed is very small compared to c, it is still possible to detect time dilation. Recall the numerical value of δ is given in the problem statement.

An experiment was fairly recently done in 2010 to indeed observe the effects of time dilation in a tabletop experiment with an atomic clock. The numbers relevant to that actual experiment were quoted above.

25 **Problem 2:** Consider the theory of Galilean relativity, as discussed in Lecture 1, where we showed that two different observers would agree that momentum is conserved in a two particle collision in one dimension. Show that if kinetic energy is conserved during this same collision in one frame, then it is also conserved in the other frame.

[1] In reality, the experimentalist does not directly measure "ticking" – this leads to complications which are important for actually carrying out the experiment, but not for understanding the essential physics being studied.

Problem 3 (Sagnac interferometer): In 1913, Georges Sagnac devised an experiment to try and disprove Einstein's theory of relativity. The experimental set-up went as follows: consider a thin circular loop (these days, of fiber optical cable), wound in a circle of radius r. Place the loop on a turntable which rotates (about the center of the loop) at angular frequency ω in the counterclockwise (CCW) direction. In this problem, you are an observer, standing at rest next to the rotating turntable. Assume $c \gg \omega r$.

Now, imagine placing (at one point on the loop) a little emitter and detector of light. It sends one pulse of light traveling through the cable in the CCW direction, and one pulse of light traveling in the clockwise (CW) direction, and detects the time it takes for each pulse to travel around the loop and hit the detector on the opposite side.[2]

15 **3A:** Let's begin by understanding what will happen.

3A.1. Using the fact that you observe light travels at a constant speed, explain why one pulse of light takes time t_1 to hit the detector, while the other takes time t_2, where

$$t_1 = \frac{2\pi r}{c - \omega r}, \tag{5a}$$

$$t_2 = \frac{2\pi r}{c + \omega r}. \tag{5b}$$

Determine which of the two times is associated with the CW pulse vs. the CCW pulse.[3]

3A.2. Was Sagnac's idea merely a thought experiment, or an actual laboratory experiment? Make educated guesses about the values of r and ω in a feasible laboratory experiment (e.g. one that could be carried out in Duane). One trick that you can use is to wind the loop on the turntable many times: with N windings, this allows you to observe the difference in "pulse times" when

$$N \times (t_1 - t_2) \geq \tau, \tag{6}$$

where the time $\tau \approx 10^{-15}$ s is sufficient.[4]

10 **3B:** Sagnac devised this experiment to try and disprove Einstein's theory of special relativity. His logic was roughly as follows:

> You, the stationary observer, see that $t_1 > t_2$. Both you and "the detector" have to agree on the order in which the CW and CCW pulses were detected. This is a consequence of causality. In the reference frame of the detector, however, relativity says there can be no spontaneous preference for CW vs. CCW motion, and the pulses must therefore arrive at exactly the same time. Thus special relativity is inconsistent with (5): the detector does not observe the speed of light as equal to the universal value c.

Explain which of the three highlighted statements above (one in red, one in orange, and one in blue) is wrong. In doing so, explain how Sagnac's experiment is actually consistent with relativity.

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[2] Actually, this is not quite how the experiment would work. You actually want to exploit the wave-like nature of light and study interference of two beams of light – but we have not yet talked about waves! The time $\tau = 10^{-15}$ s is roughly the oscillation time of visible light.

[3] *Hint:* Write down a formula for the angular position (as a function of time) for the CW pulse, the CCW pulse, and the detector on the rotating table. Remember that (in radians) the angle θ is physically the same as $\theta + 2\pi$.

[4] The reason why such a short time is possible to observe has to do with the wavelike nature of light, which "pulses" on this time scale, and phenomena of wave interference. We will learn about the physics of waves later in this class.

20 **Problem 4 (Atmospheric muons):** Cosmic rays strike the Earth's atmosphere and create muons, elementary particles which then stream towards the Earth's surface where they can be detected, at a distance $x \approx 10^4$ m from where they were created. The speed of the muons is approximately $v \approx 0.98c$. In its rest frame, a muon has a mean lifetime of $\tau \approx 2 \times 10^{-6}$ s. The probability that a muon has not decayed by time t (in the rest frame of the muon) is given by

$$P(t) = \mathrm{e}^{-t/\tau}. \tag{7}$$

4.1. How long does it take for a muon to hit the surface of the Earth, as measured by someone standing at rest on Earth's surface? Call this time t_1.

4.2. If we attached a little clock to the muon when it was created, what is the amount of time elapsed on that clock when the muon hit the surface of the Earth? Call this amount of time t_2.

4.3. Calculate $P(t_1)$ and $P(t_2)$ and give the numerical values for each.

4.4. Which of the two probabilities found above represents the actual physical probability of detecting any given muon on the surface of the Earth? Why?

Experiments in the 1940s were able to confirm directly that the muon detection rate matched the predictions of special relativity.

15 **Problem 5 (Apparent superluminal stars):** There is an "optical illusion" which suggests that objects are moving faster than light – best illustrated by thinking about the motion of stars in the night sky. Suppose that star X is moving with velocity vector

$$\mathbf{v} = c\,(\beta_x\hat{\mathbf{x}} + \beta_y\hat{\mathbf{y}}) \tag{8}$$

You, the observer, are located at the point $0\hat{\mathbf{x}} + 0\hat{\mathbf{y}}$. Note that $\sqrt{\beta_x^2 + \beta_y^2} < 1$. Suppose that at time $t = 0$, the star is at position

$$\mathbf{r}(0) = D\hat{\mathbf{x}}. \tag{9}$$

Call this spacetime point event A.

5.1. At time t, what spatial position is the object at: $\mathbf{r}(t)$? In this problem, you may assume t is small if this approximation helps you. Call the correspoding spacetime event $B = (t, \mathbf{r}(t))$.

5.2. At event A, a light pulse is sent towards you. At what time t_1 will it reach you?

5.3. At event B, another light pulse is sent towards you. At what time t_2 does it reach you?

5.4. What is the relative angular difference θ between the directions from which the first and second pulses were sent? (I would use the convention $\tan\theta = y/x$ here.)

5.5. Suppose that, you, the observer, assume that the star was moving around you in a *circle* of radius D. Then you would see an *apparent velocity*

$$v_{\mathrm{ap}} = \frac{D\theta}{t_2 - t_1}. \tag{10}$$

Justify this equation.

5.6. Now show that $v_{\mathrm{ap}} > c$ is possible.

Of course, superluminal propagation is not actually possible – the point is that relativity can lead to optical "illusions" (or perhaps more accurately, "delusions") if we don't very carefully keep track of notions of space and time. We will learn many more examples of this!