PHYS 2170: General Physics 3 for Majors **Fall 2021**

Homework 11

Due: December 2 at 11:59 PM. Submit on Canvas.

Problem 1 (Rydberg atoms): Consider an electron in an orbit of the hydrogen atom with a very large value of n (i.e. a state bound less strongly to the proton).

25 **1A:** Let us begin by reviewing the Bohr quantization argument, which is based on finding semiclassical circular orbits of hydrogen.

1A.1. If the electron is moving in a classical circular orbit of radius r, what must its velocity v be?

1A.2. If angular momentum is quantized to be $L = n\hbar$, show that the radius of the n^{th} orbit takes the form

$$r_n = n^2 a_{\text{B}}, \tag{1}$$

and find an expression for a_{B}, along with its numerical value.

10 **1B:** Using the ideal gas law, we can estimate that at room temperature and pressure, the typical spacing between atoms is given by d, where

$$\frac{1}{d^3} = \frac{P}{k_{\text{B}}T}, \tag{2}$$

where P is pressure ($\sim 10^5$ Pa, the usual SI unit, at ambient conditions), T is temperature (~ 300 K at room temperature), and $k_{\text{B}} \approx 1.4 \times 10^{-23}$ J/K.

If the radius r_n of orbit is comparable to d, then we cannot think of the electrons as strongly bound to an individual protons – instead, one electron will strongly interact with multiple protons. Such a strongly interacting quantum system has many valuable properties which make these "Rydberg atoms" a current research area, including at JILA in Boulder.

1B.1. Calculate d at room temperature and pressure.

1B.2. At these conditions, for what value of n will we find r_n comparable to d?

1B.3. What pressure would we need to lower a gas of hydrogen atoms to to find $n = 100$?

25 **Problem 2:** Consider a particle moving on a two dimensional ring, whose energy levels are given in terms of angular momentum by

$$E = AL_z^2 - BL_z. \tag{3}$$

Here $A, B > 0$ are constants.

2.1. What are the allowed values of angular momentum L_z?

2.2. What are the allowed energy levels of this system?

2.3. Describe what happens to the ground state of the system as the parameter B is increased from 0.

Problem 3 (Rotation of molecules): Consider the flourine molecule F_2, which consists of two flourine atoms of mass $m \approx 3 \times 10^{-26}$ kg, separated by $a \approx 1.4 \times 10^{-10}$ m.

10 **3A:** Calculate the classical moment of inertia I about the center of this molecule, assuming that the H atoms lie in the xy-plane and that the rotation is about the z-axis.

10 **3B:** The rotation of this molecule can be described by the energy

$$E = \frac{\mathbf{L}^2}{2I}. \tag{4}$$

3B.1. What are the allowed values of the angular momentum $\mathbf{L}^2$ in quantum mechanics?

3B.2. What are the allowed rotational energy levels in the flourine molecule?

15 **3C:** A photon can be absorbed or emitted by this molecule, changing the "total angular momentum" quantum number l by either ± 1.

3C.1. Deduce the allowed wavelengths of light which can be absorbed by changing the rotational quantum state of the flourine molecule.

3C.2. What is the longest wavelength of light which can be absorbed? Is it visible light?

5 **3D:** At room temperature, we might expect energy states to be occupied so long as their energy $E \lesssim 4 \times 10^{-21}$ J.

3D.1. Estimate the total number of angular momentum states that might be occupied at room temperature. Make sure to account for any degeneracy in the energy levels in (4)!

3D.2. When the quantum number $l \gg 1$, it is reasonable to approximate that the rotation of the flourine molecule is classical. Is this a reasonable approximation at room temperature?

Problem 4 (Relativistic corrections in atomic physics): In this problem, we will try to correct Bohr's model for energy levels in the hydrogen atom (and beyond) to account for relativistic corrections.

5 **4A:** First, we need to determine the first relativistic correction to the non-relativistic kinetic energy. Show that the kinetic energy of a particle is approximately given by

$$K \approx \frac{mv^2}{2} + \frac{3mv^4}{8c^2}. \tag{5}$$

10 **4B:** We crudely estimate that if the electron is in the n^{th} Bohr orbit, it's energy level is adjusted by

$$\Delta E = \Delta K_{\text{rel}}, \tag{6}$$

with ΔK_{rel} the relativistic correction found above.

4B.1. How large is the correction to the n^{th} energy level of hydrogen due to fine structure? Express your answer in terms of the magnitude of the ground state energy (call it E_0), and the **fine structure constant**

$$\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c} \approx \frac{1}{137}. \tag{7}$$

4B.2. What is the wavelength of the photon emitted by an electron transitioning from the $n = 2$ to $n = 1$ states of hydrogen, if we account for the relativistic correction to the energy levels that we estimated above? How much will this wavelength differ from the non-relativistic answer?

5 **4C:** At room temperature, the typical velocity of a hydrogen atom would be about 1400 m/s. Is the correction found in the previous part distinguishable from the smearing of the spectrum of the hydrogen atom due to the Doppler effect?

5 **4D:** Let us now consider the energy levels in the uranium atom, which has $Z = 92$ protons in the nucleus. Consider electrons bound tightly to this nucleus in the $n = 1$ level.

4D.1. Explain how to modify the energy levels and velocities found in Bohr quantization when the nucleus has charge Ze.

4D.2. Are relativistic corrections to the ground state energy negligible in the uranium atom?[1]

[1] The answer to this, in part, has some crude implications for whether relativity is ever important in chemistry!