PHYS 2170: General Physics 3 for Majors **Fall 2021**

Homework 2

Due: September 9 at 11:59 PM. Submit on Canvas.

25 **Problem 1 (Detecting elementary particles):** A particle collider (such as the LHC in Geneva) works by smashing together beams of oppositely moving particles which are moving close to the speed of light; after a collision, many exotic particles might be created. One such particle which was detected in 2012 is the Higgs boson, the last part of the Standard Model to be discovered in experiment.

Suppose that after such a collision, a Higgs boson is created and travels at velocity $v = c\beta$ (as viewed by an experimentalist) towards a detector a distance L away. Because the Higgs is unstable, it will decay after a time τ into other elementary particles.

1.1. Explain, using the reference frame of the experimentalist, why they can detect the Higgs directly so long as

$$\tau \geq \frac{\sqrt{1-\beta^2}}{\beta}\frac{L}{c}. \tag{1}$$

1.2. Explain this result from the Higgs' reference frame – what relativistic effect leads to (1)?

1.3. At LHC, particles travel at speeds very close to c, so we can write $\beta = 1 - \delta$, where $\delta \ll 1$ is a very small number. Argue that we can then estimate

$$\frac{\sqrt{1-\beta^2}}{\beta} \approx \sqrt{2\delta}. \tag{2}$$

1.4. The Higgs boson has lifetime $\tau \approx 10^{-22}$ s. The particle detection chambers at LHC actually aim to track the trajectories of particles rather than collecting them at a detector, so we can estimate $L \approx 10^{-5}$ m is the resolution that the detectors might reach. Particles travel at speeds where $\delta \approx 10^{-8}$. Could the experimentalists directly "detect" a Higgs boson, or must they have "seen" it indirectly (e.g. by measuring what it decays into)?

20 **Problem 2:** Consider a futuristic spaceship flying past the Earth – which is approximately a sphere – at a fast and constant velocity $v = c\beta$.

2.1. Qualitatively describe how the Earth will look to an observer on the spaceship.

2.2. Let V be the volume of the Earth, as measured by an observer on the surface of the Earth (which you can regard as approximately an inertial frame). Find a formula for V', the volume measured by someone on the moving spaceship, in terms of V and β.[1]

2.3. Find the value of β at which the Earth's volume is measured to be $V/2$ by someone on the spaceship.

[1] *Hint:* As a thought experiment, imagine that the spaceship is moving in the x-direction. Suppose that there were three giant rulers, of length the diameter of Earth (in Earth's frame), aligned along the x, y, and z axes and passing through the center of the Earth. How long would each ruler appear on the spaceship?

15 **Problem 3:** Consider two events A and B which take place somewhere in spacetime. Suppose that an observer in frame R sees that the relative time difference between A and B is Δt. Now consider another observer in inertial frame S, which moves at velocity $v \neq 0$ relative to R. Is it possible for S to also see time delay Δt between A and B? If you answer yes, show how this can happen with an explicit example; if you answer no, show why it cannot happen.[2]

20 **Problem 4 (Would we see time dilation?):** Consider the thought experiment of Lecture 2, where we imagined observer Blue witnessing a clock moving at a velocity v. Suppose that in Blue's coordinate system, Blue is located at $x = 0$, while at time $t = 0$ (as measured in coordinate time in Blue's frame), the clock is at $x = D > 0$ and is moving with velocity v. Suppose that at this event $(t, x) = (0, D)$, the clock ticks and sends a signal, at the speed of light, towards Blue.

4.1. At what coordinate time T_0 does Blue receive the signal?

4.2. Now suppose the clock ticks after a time interval of τ in its own rest frame; assume that $D \gg c\tau$. What are the spacetime coordinates of the event (t, x) at which the *next* tick happens?

4.3. Again, a signal is sent towards Blue at speed c. At what time T_1 does Blue receive it?

4.4. The actual ticking time that Blue *observes* is $\Delta T = T_1 - T_0$. Compare ΔT to our formula for time dilation from in class, and comment on what you find. Is it possible for $\Delta T < \tau$? If so, "what happened to time dilation"?

20 **Problem 5:** Consider two runners A and B, who start at $x = 0$ and $x = D$ respectively at time $t = 0$ in frame S. The runners agree that when frame S's clock hits $t = 0$, they will begin to run at a constant velocity v (in the x-direction). (Assume the time spent accelerating to that velocity is negligible). Let frame S' (which agrees with frame S on the origin of spacetime) move at constant velocity v relative to S, such that (for times $t' > 0$) runner A sits at $x' = 0$.

5.1. Show that in frame S, the 2 runners are always a distance D apart.

5.2. Show that in frame S' (namely, as viewed by the "moving" A), the distance between the two runners is γD.

5.3. Explain from runner A's perspective, as simply as possible, why the distance to B increased.

20 **Problem 6:** Consider three events A, B and C in spacetime, and three inertial reference frames R, S and Q. You may assume that "observers" in the three frames agree on the event $t = x = 0$ (namely, the origin of spacetime), but may be moving at different velocities (which are oriented in the x direction). Denote the coordinates of event A in inertial reference frame R with the coordinate pair $(t_{A,R}, x_{A,R})$.

Show that the following scenario is mathematically forbidden: in frame R, $t_{A,R} < t_{B,R} < t_{C,R}$, while in frame S, $t_{B,S} < t_{C,S} < t_{A,S}$, while in frame Q, $t_{C,Q} < t_{A,Q} < t_{B,Q}$. Note that these inequalities are strict!

[2] *Hint:* In frame R, you can pick any points (t, x) for A and B, and then define $\Delta t = t_B - t_A$. What will the spacetime points of A and B be in frame S?