PHYS 2170: General Physics 3 for Majors **Fall 2021**

Homework 6

Due: October 14 at 11:59 PM. Submit on Canvas.

Problem 1 (Flute): A flute is a wind instrument that, crudely, works as follows. The player blows into a metal tube of length ~ 1 m at one end. The tube is best modeled as "open at both ends" (despite appearances!). The player can open or close up holes in the tube that effectively shorten the length of the tube (see Figure 1). In this way, the player can create sound at different frequencies, i.e. musical notes.

15 **1A:** Let's begin by plugging in some numbers.

- 1A.1. Let c denote the speed of sound in air. If L_n is the length of the tube when note n is played (i.e. a certain hole is opened), find the fundamental frequency of the musical note that will be played.
- 1A.2. If $c \approx 340$ m/s, estimate the lowest frequency note that could be played by a flute, and compare to the frequency of a typical musical instrument (~ 400 Hz).
- 1A.3. In a cold room, the speed of sound can easily drop by ~ 5 m/s. Estimate the ratio of the fundamental frequency of the flute in a cold room, relative to a "normal" room.

10 **1B:** In an orchestral setting, a flute whose frequencies are offset by $\sim 1\%$ could sound quite unpleasant – the human ear is very sensitive to hearing two instruments with slightly offset frequencies of this order. (The piano's frequencies would not change as drastically as the flute's in a cold room.) A common mechanical feature that a Western concert flute has is the ability to adjust the length of the tube, as shown in Figure 2.

- 1B.1. Explain how this can help to correct for variable air temperature and help the flute play the correct notes.
- 1B.2. Could a fixed adjustment of total tube length lead to *every* note staying exactly in pitch?

Figure 1: Pueblo flute from Arizona State University: the uncovered hole determines the fundamental frequency of sound played.

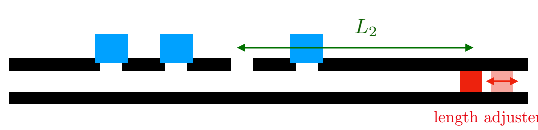

Figure 2: A sketch of how a concert flute works; the blue keys block off most air holes. The red length adjuster can be used to try and correct the frequency of notes played.

Problem 2: Consider a triangular shaped pulse, as depicted in Figure 3, which propagates to the left along a string towards a boundary at $x = 0$. Assume the speed of waves on the string is $v = 1$ m/s.

15 **2A:** Sketch $y(x, t)$ at $t = 0, 1/3, 2/3, 1, 4/3$ s, assuming that the string is tied down at $x = 0$. Explain your answer.

15 **2B:** Sketch $y(x, t)$ at $t = 0, 1/3, 2/3, 1, 4/3$ s,assuming that the string is loosely held (free boundary condition) at $x = 0$. Explain your answer.

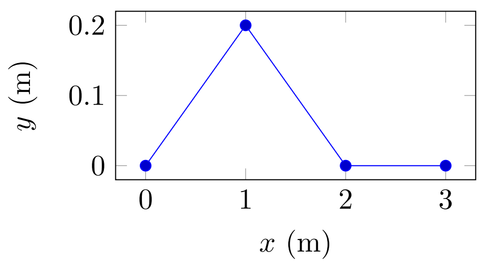

Figure 3: Wave profile $y(x, t = 0)$.

15 **Problem 3:** Expand out the real and imaginary parts of $\mathrm{e}^{\mathrm{i}(a+b)} = \mathrm{e}^{\mathrm{i}a}\mathrm{e}^{\mathrm{i}b}$ where a and b are real. Show that you obtain the trigonometric identities for sine and cosine addition by using Euler's formula on each complex exponential in this equality.

20 **Problem 4 (Waves on a circle):** Consider the wave equation

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2} \tag{1}$$

on a periodic domain (a circle). A mathematical way to implement this is to demand that the solution $y(x, t)$ to this differential equation obey (for all x and t) $y(x, t) = y(x + L, t)$, where L is the circumference of the circle.

4.1. Plug in the ansatz $y = Y(x)\mathrm{e}^{-\mathrm{i}\omega t}$ and derive a differential equation for $Y(x)$.

4.2. Try to solve this differential equation using a standard ansatz, and determine the allowed form(s) of $Y(x)$ in terms of ω, v and x.

4.3. Show that imposing $Y(x) = Y(x + L)$ constrains the values of ω to

$$\omega = \frac{2\pi n v}{L},; \ n = 0, \pm 1, \pm 2, \ldots. \tag{2}$$

This describes the normal modes of waves on a circle.

Problem 5: Consider a pendulum which consists of a string of *total mass* m and length L, tied rigidly to the ceiling at one end, and to a mass M at the other end. The pendulum hangs in gravity.

15 **5A:** Let's begin by assuming that $M \gg m$.

5A.1. Reviewing your earlier mechanics course, determine the frequency of oscillations of the pendulum, if the mass is swinging.

5A.2. Now, suppose that you instead plucked the string by which the pendulum hangs. What is the fundamental frequency of oscillations of the string?[1]. Compare to the answer from 5A.1.

5 **5B:** In a more advanced physics class, you would learn how to solve this problem for the opposite limit of $M \ll m$ – in particular, taking $M = 0$, we would find the vibration frequency of the hanging string on its own. Can you explain why the fundamental frequency of the hanging string by itself cannot depend on m?

[1] *Hint:* Assume that the string is held fixed at both ends. What are the tension and mass per unit length of the string?

15 **Problem 6 (Speed of sound in a composite solid):** Consider a composite solid, as depicted in Figure 4, which consists of alternating layers of materials 1 and 2, which have different Young's modulus $Y_{1,2}$ and densities $\rho_{1,2}$.

6.1. What is the *average* mass density of this composite solid?

6.2. What is the *effective* value of the Young's modulus, if we compress the solid in the x-direction?[2]

6.3. What is the *effective* value of the Young's modulus, if we compress the solid in the y-direction?[3]

6.4. Given the list of material properties in Figure 5, design a material with the lowest speed of sound possible, for waves propagating in the y-direction. What are your guiding design principles? (You can use the same material twice if you want.)

6.5. Repeat the previous part, but now try to maximize the speed of sound.

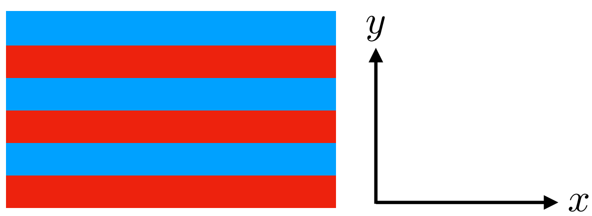

Figure 4: Structure of a composite solid made up of materials 1 and 2.

	Y (GPa)	ρ (kg/m^3)
aluminum	70	2700
diamond	1100	3500
concrete	30	2500
polycarbonate	2	1200
rubber	0.1	1200

Figure 5: A table with the properties of some "common" solids.

[2] *Hint:* If the solid is glued together, then the displacement $u(x,t)$ has to be the same in materials 1 and 2.

[3] *Hint:* In equilibrium, the net forces on the boundary between materials 1 and 2 must vanish. Use this to determine the relative amount of displacement between slabs 1 and 2.