PHYS 2170: General Physics 3 for Majors **Fall 2021**

Homework 7

Due: October 21 at 11:59 PM. Submit on Canvas.

25 **Problem 1 (Ambulance siren):** While stuck in traffic (and approximately at rest) you hear an ambulance siren. The ambulance is first approaching from behind you, and after it passes, you notice that the frequency of the siren (as heard by you) has decreased by 10%.

Given the speed of sound in air is about 340 m/s, how fast is the ambulance driving?

Problem 2 (Vibrations of a bridge): Consider a simple pedestrian overpass bridge over a highway, made out of concrete with mass density $\rho = 2400$ kg/m^3, Young's modulus $Y = 40$ GPa, and thickness $w \sim 2$ m. The bridge spans a length of $L \sim 50$ m over a highway, and is supported on each end but has no supports over the middle of the road. As people walk across the bridge, their feet will hit the pavement at a frequency of $f_* \sim 1$ Hz. Might this walking be structurally dangerous for the bridge? We should worry that it could be if f_* is reasonably close to the fundamental frequency of one of the normal modes of the bridge itself.

In this problem, you do not need to worry about calculating things to within more than an order of magnitude of the right answer. The goal is to make sensible estimates about a real world problem using only basic physics knowledge!

10 **2A:** Let's begin by thinking about the longitudinal sound waves along the bridge.

- 2A.1. Determine the speed of sound waves in concrete using the numbers above.
- 2A.2. Estimate that the longest wavelength which is resonant has wavelength $2h$. What is the frequency of this mode? Compare to f_* and comment.

10 **2B:** Now let's consider the flexural waves along the bridge. Following Lecture 20, estimate the fundamental frequency of flexural modes on the bridge. Compare to f_* and comment.

10 **2C:** In order to make the flexural modes have higher frequencies, do you think it is more efficient to try and make the bridge thicker or to add a support halfway along the bridge's span?

Problem 3: Consider an observer at rest in a frame where the ambient air is also at rest. Far away to the left, a speaker emits sound waves at angular frequency ω_0; far away to the right, an identical speaker (emitting the same frequency waves in its rest frame) is moving at velocity u to the right. Let v denote the speed of sound in air.

10 **3A:** Assuming the amplitude of waves from each speaker is the same, we might expect that the (complex) amplitude of sound waves is given by (for all points x between the two speakers)

$$u(x,t) = A\left(\mathrm{e}^{\mathrm{i}k(x-vt)} + \mathrm{e}^{\mathrm{i}k'(x+vt)}\right). \tag{1}$$

Here A is an unimportant overall constant prefactor. Use the Doppler effect (if needed), and other physics of waves, to determine k and k' in terms of u, v and/or ω.

10 **3B:** If the observer is at $x = 0$, the amplitude of oscillations is given by $u(0, t)$. Show that if $u \ll v$, then[1]

$$\mathrm{Re}[u(0, t)] = 2A\cos(\omega_1 t)\cos(\omega_2 t) \tag{2}$$

where $|\omega_2| \ll |\omega_1|$. Find explicit formulas for $\omega_{1,2}$ in terms of ω_0, u, v. $\cos(\omega_2 t)$ varies extremely slowly in time. This leads to **beating** – two waves interfering at slightly different frequencies will lead to a slow variation in the amplitude with time. Beating waves are particularly audible, which makes music that is even slightly out of tune very unpleasant.

Problem 4 (Tsunami): Tsunamis are waves on the surface of the ocean, generally created by earthquakes. They have notoriously long wavelengths of $\lambda \sim 100$ km. The mass density of water is $\rho \approx 1000$ kg/m^3, and the acceleration due to gravity is $g \approx 10$ m/s^2.

10 **4A:** Suppose we *estimate* a formula for the velocity of water waves of the form

$$v \approx \rho^a g^b \lambda^c, \tag{3}$$

where a, b, c are numbers. Fix a, b and c by demanding that v has units of m/s. Note that you cannot fix whether or not there should be a factor of 2 or π (e.g.) in (3) – usually in physics, the missing constant is not too far off from 1, so you can get away with making quick estimates like this.

10 **4B:** You may notice that v is *not* linearly proportional to wave number $k = 2\pi/\lambda$. Therefore, might we have to worry about whether v is a phase velocity or a group velocity? Argue that if v found above represented the *phase velocity* of the waves, the group velocity would only differ by a factor of 2.[2]

5 **4C:** The 2011 Tohoku earthquake in Japan spawned a tsunami which, 8 hours later, hit the coast of California. If the distance between Japan and California is about 9000 km, use your answer from above to predict the time it would take for the tsunami to reach California; compare to observation.[3]

Problem 5: Consider a string of mass per unit length μ and charge per unit length η, held at tension T and oriented in the x-direction. The string is free to vibrate in both the y- and z-directions.

5 **5A:** Argue that small displacements $y(x, t)$ and $z(x, t)$ describing the motions of the string in the two transverse directions obey

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}, \tag{4a}$$
$$\frac{\partial^2 z}{\partial t^2} = v^2 \frac{\partial^2 z}{\partial x^2}, \tag{4b}$$

for an appropriate wave velocity v which you should find.

15 **5B:** Now suppose we turn on a magnetic field $\mathbf{B} = B_0\hat{\mathbf{x}}$.

5B.1. How do the wave equations above get modified?

5B.2. Define the variables

$$w_\pm = y \pm \mathrm{i}z. \tag{5}$$

Show that $w_\pm(x, t)$ obey *decoupled* differential equations (i.e. you get one equation for w_+ and one for w_-).

[1] *Hint:* Use $\mathrm{e}^{ia} + \mathrm{e}^{ib} = \mathrm{e}^{\mathrm{i}c}(\mathrm{e}^{\mathrm{i}(a-c)} + \mathrm{e}^{\mathrm{i}(b-c)})$ in a clever way to avoid trig addition identities.

[2] As we can't fix the constant prefactor of (3), we should not be concerned about whether (3) is phase or group velocity.

[3] Your answer is probably a tiny bit off – the reason is that the formula (3) is not perfectly accurate for the long wavelength of a tsunami.

5B.3. Using the ansatz $w_\pm = \mathrm{e}^{\mathrm{i}kx - \mathrm{i}\omega t}$, deduce the dispersion relation $\omega_\pm(k)$ for the modes $w_\pm$.

5B.4. Do w_+ and w_- each represent a physically distinct mode?

5B.5. Comment on the modes of the string in the $k \to 0$ limit. Does your answer make physical sense?