Homework 8

Due: October 28 at 11:59 PM. Submit on Canvas.

25 **Problem 1 (Soap bubbles):** We have all seen colorful soap bubbles (I hope) as children. The interesting patterns of color that arise in the bubble are a consequence of ambient light passing through the soap film, reflecting or transmitting off of the boundaries between soap and air, and constructively (or destructively) interfering. Let d denote the unknown soap bubble film thickness. The index of refraction of air $n_{\text{air}} \approx 1$, while the index of refraction in the soap film is $n \approx 1.3$.

1.1. Determine the criterion on the soap film thickness d in order to have constructive interference between incident and reflected light of wavelength λ. Assume it is incident normally on the soap film.

1.2. Visible light has $\lambda \approx 600$ nm in air. Given that visible light can constructively interfere on the soap film, estimate its thickness.

Problem 2 (Expansion of the universe): The universe is inherently expanding all around us! The effect is small on Earthly scales but is not small on astronomical scales. **Hubble's Law** tells us that a star "at rest" a distance r from us on Earth will actually appear to be moving *away* from us at a speed given by

$$v = Hr \tag{1}$$

where $H \approx 2 \times 10^{-18}\ \text{s}^{-1}$ is the **Hubble constant**.

10 **2A:** Recall that no object can travel faster than the speed of light, c.

2A.1. Argue that there is a finite radius R to the visible universe. What is the numerical value?

2A.2. Suppose we were to observe the light emitted by an event a distance $r < R$ away from us on Earth. How far in the past (in our reference frame) did this event happen?

15 **2B:** One modern source of astronomical data is electromagnetic radiation at wavelength 21 cm, which arises from hydrogen atoms in the interstellar medium.

2B.1. Combine Hubble's Law with the Doppler effect to conclude that an astronomer could determine the distance r to some "event" if they could measure the *observed wavelength* of this 21 cm radiation on Earth. Determine the relationship between the observed wavelength λ of light, and the distance r to an astronomical object.

2B.2. Combine the previous parts of this problem to conclude that by studying the 21 cm radiation at different wavelengths (as detected by the Earth observer), we can gain a view into events that happened at different times in the universe. Estimate how far back in time we can see, and compare to the age of the universe ($\approx 10^{10}$ years).

In reality, this problem is a serious oversimplification. Since stars are moving away at an increasing rate, the universe does not consist of inertial frames! To do this problem correctly, we have to consider corrections from general relativity which are beyond the scope of this course.

Problem 3 (Data storage): An old-school (1990s) compact disc (CD) stores data by etching bits of data (0 or 1) into tiny grooves in the highly reflective disk, and then using carefully focused laser light of wavelength $\lambda \sim 600$ nm to read off the etched pattern in the disc.

10 **3A:** For this to properly operate, the grooves and etches on the CD must be able to serve as a diffraction grating for the laser light.[1] Argue that the grooves on the device must have a width $w \geq \lambda$.

10 **3B:** What are the consequences of this for data storage on the disc?

- 3B.1. Using real world knowledge of the typical size of a CD or similar disc, estimate the available surface area on the disc which can be used for storage.
- 3B.2. Being generous and estimating that we can store a single bit in a surface area of w^2, how many bits can we store in the disc?
- 3B.3. A byte has 8 bits. Estimate how many bytes (B) of data can be stored on the CD. The actual answer is usually a bit less than 1 GB (the prefactor is giga, like in SI units); are you close?

5 **3C:** Around 10 years ago, Blu-Ray was developed. Blu-Ray discs can only be read using blue laser light. Explain why these discs can have superior storage capabilities over the older CDs.

Problem 4: Consider the 3 slits (of negligible thickness) depicted in Figure 1. Suppose that there are three possible sources of coherent light, located a far distance away from the slit at incident angles of 0 (X), $-\alpha$ (Y) and $+\alpha$ (Z). Let k denote the wave number of the incident light.

10 **4A:** Write a formula for the (relative) intensity of light observed at angle θ far away from the slits, when only source X is turned on.

10 **4B:** Repeat the calculation when only source Y is turned on. Comment on your answer.

5 **4C:** Repeat the calculation when both Y and Z are turned on. You can assume that sources Y and Z are in phase with each other, and have the same amplitude.

20 **Problem 5:** Consider a triangle-shaped slit (of negligible thickness) in a two-dimensional plane, as shown in Figure 2. An incident plane wave of light shines directly on the slit. Describe the pattern of light intensity seen on a screen a large distance away from the triangle, and explain your result.[2]

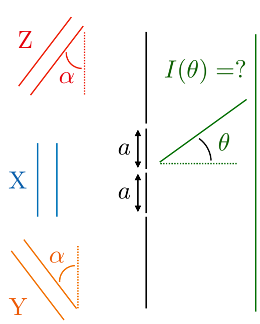

Figure 1: Three sources of light illuminate 3 narrow slits.

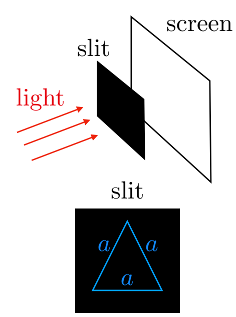

Figure 2: Illumination of a triangular slit.

[1] If it could not, then imaging the reflected light would not lead you to detect any tiny variations on the surface.

[2] *Hint:* Approach this problem in two steps. First, let $(X(s), Y(s))$ be a (piecewise smooth) vector function which traces out the triangle's shape on the screen. Generalize the method of Lecture 26 to a three-dimensional problem. You should use `Mathematica` (e.g.) to carry out integrals and plot the resulting intensity distribution.