PHYS 2170: General Physics 3 for Majors **Fall 2021**

Homework 9

Due: November 11 at 11:59 PM. Submit on Canvas.

Problem 1 (Sunscreen): We are told to wear sunscreen when going outside for long periods of time – *especially* at high altitude in Colorado! Why?

10 **1A:** The mass of an electron is $m \approx 10^{-30}$ kg, and the size of a chemical bond is about $L \sim 0.1$ nm.

1A.1. We might estimate that the energy stored in a chemical bond is

$$E \sim \hbar^a m^b L^c. \tag{1}$$

Find the values of a, b and c for which the units on each side of this equation match.

1A.2. Plug in appropriate values into this formula and estimate the energy E_* stored in a chemical bond, in Joules.

15 **1B:** Suppose that a photon of wavelength λ hits this bond. If it has enough energy to break the bond by injecting energy E_* into the electrons, that can cause biological damage.

1B.1. There is a critical value of λ_c, either above/below which the photon does not have enough energy to break the chemical bond. What is λ_c?

1B.2. Should wavelengths be above or below λ_c in order to break the bond?

1B.3. Can you explain why sunscreen is intended to primarily reflect UV light?

Problem 2 (Mesons): Mesons are elementary particles made out of a single quark-antiquark pair. Assume that each quark and anti-quark is approximately a massless relativistic particle. We can model them as being connected by a "string" of constant tension force T. For simplicity in your model below, treat the quark as stationary and consider only the motion of the antiquark around the quark.

10 **2A:** Argue that the combined kinetic and potential energy of the antiquark is given by

$$E = c|p| + T|x|, \tag{2}$$

where x denotes the distance of the quark to the antiquark, and p denotes the momentum of the antiquark.

15 **2B:** What is the quantum mechanics of this system?

2B.1. Use Heisenberg's uncertainty principle to estimate the size of the quark (i.e. the value of x at which energy is minimized).

2B.2. A meson has a typical mass $m \sim 10^{-28}$ kg. Use $E = mc^2$ to deduce the magnitude of the tension force T that holds the quark together.

2B.3. Determine the numerical value of the size of a meson.

20 **Problem 3 (Size of a nucleus):** Consider a helium-4 nucleus, which has 2 protons and 2 neutrons bound together quantum mechanically. It is known that it takes about 7 MeV (or about 10^{-12} J) of energy to eject a neutron (e.g.) from this nucleus.

3.1. If a neutron of mass $m \approx 2 \times 10^{-27}$ kg has an energy of 7 MeV, what is the velocity of that nucleus?

3.2. Deduce the quantum mechanical wavelength of this neutron.

3.3. A crude estimate for the size of the nucleus is that it is the wavelength of the neutron with this binding energy. Compare your estimate for the size of a nucleus to the actual size of about 10^{-15} m.

Problem 4: Let $a, b > 0$. Consider a quantum state in one dimension with wave function

$$\Psi(x) = A\mathrm{e}^{\mathrm{i}bx}\mathrm{e}^{-a|x|}. \tag{3}$$

10 **4A:** Let's learn a neat trick for calculating some integrals. For $c > 0$, let

$$I_n = \int_0^\infty \mathrm{d}x\; x^n \mathrm{e}^{-cx}, \tag{4}$$

4A.1. What is I_0?

4A.2. Treat the number c as a variable, and show that for any n,

$$\frac{\partial I_n}{\partial c} = -I_{n+1} \tag{5}$$

4A.3. Defining the factorial $n! = 1 \times 2 \times \cdots \times n$ (with $0! = 1$), show that

$$I_n = \frac{n!}{c^{1+n}}. \tag{6}$$

5 **4B:** Demand that the wave function is normalized, and fix the constant A.

10 **4C:** What is the spatial spread of this wave function?

4C.1. Calculate $\langle x \rangle$.

4C.2. Calculate $\langle x^2 \rangle$.

4C.3. Calculate Δx.

10 **4D:** What is the uncertainty in momentum of this wave function?

4D.1. Calculate $\langle p \rangle$.

4D.2. Calculate $\langle p^2 \rangle$.

4D.3. Calculate Δp.

4D.4. How close does this wave function come to saturating the Heisenberg uncertainty principle?

15 **Problem 5:** Consider a thin needle of total mass m and length L. If it has an "atomically thin" endpoint but is otherwise macroscopic in size, you would not expect to be able to stand the needle exactly on its endpoint. Intuitively, the reason why is that in reality you will not orient the needle such that it's center of gravity lies *exactly* above the tip of the needle, and as long as this center of gravity is even a little bit displaced, it will cause the needle to fall over. In classical physics, however, it's at least possible *in principle* to achieve this.

We will argue, however, that in quantum mechanics, even this is not possible due to Heisenberg's uncertainty principle. Treat the needle as a rigid rod, and note that Heisenberg's uncertainty principle for angular momentum L and angular displacement θ is

$$\Delta L \cdot \Delta\theta \geq \frac{\hbar}{2}. \tag{7}$$

5.1. Describe the classical motion of the needle when $|\theta| \ll 1$ (here you should define $\theta = 0$ to be when the needle is perfectly upright). Show that the needle will begin to tip over exponentially quickly if it deviates even slightly from $\theta = 0$. In your derivation, you can assume there's friction where the needle connects to the ground, such that the needle rotates rigidly about its endpoint.

5.2. Use (7) to estimate the longest possible time it might take for a needle to spontaneously fall over. Give your answer in seconds, assuming $m = 1$ g, $g \approx 10$ m/s^2, and $L \approx 10$ cm.