PHYS 2170

General Physics 3 for Majors

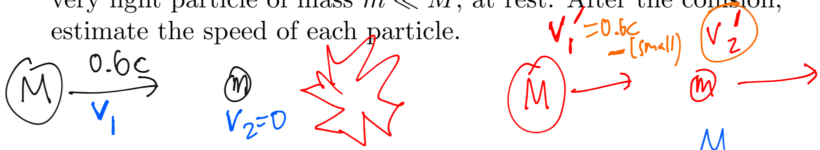
Fall 2021

Lecture 10

Nuclear fission and fusion

September 15

1 A very heavy particle of mass M, traveling at speed $0.6c$, collides into a very light particle of mass $m \ll M$, at rest. After the collision, estimate the speed of each particle.



center of mass (momentum) frame: $p_{tot} = 0 = \gamma_1 M v_1 + \gamma_2 m v_2$

$|v_1| \ll |v_2|$ . . . $v_1 \approx 0$ $\quad v_2 = -0.6c$

bef: ① → ← ②

$p_1 + p_2 = 0$

aft: ← ① ② →

$(-p_1) + (-p_2) = 0$

heavy: $E \approx Mc^2 + \frac{p^2}{2M}$

Back to old frame: $v_1' \approx 0.6c$

$$v_2' \approx \frac{v_2'^{\,COM} + 0.6c}{1 + \frac{1}{c^2}(0.6c)\,v_2'^{\,COM}} = \frac{3/5 + 3/5}{1 + (3/5)^2}\,c = c\,\frac{6/5}{1 + 9/25} = c\,\frac{30}{34} = \frac{15}{17}\,c$$

2 Why do we think about mass in units of MeV/c^2 in particle or nuclear physics? What is the mass of a neutron in these units?

$$E^2 = (cp)^2 + (mc^2)^2$$

1 MeV = energy need to push 1 electron up 10^6 V

$[U = qV]$

"if $c = 1$" [choice of units] [replace 1 m → 1 light-second]

$$E^2 = p^2 + m^2$$

E: 1 MeV = 1.6×10^{-13} J

p: 1 $\frac{\text{MeV}}{c}$ = 5.3×10^{-22} N·s

m: 1 $\frac{\text{MeV}}{c^2}$ = 1.8×10^{-30} kg

particle	mass (kg)	$\frac{\text{MeV}}{c^2}$
electron	10^{-30}	0.5
neutron	2×10^{-27}	900

$1 \frac{\text{MeV}}{c^2}$

$mc^2 = 1\,\text{MeV} \frac{1}{c^2} \cdot c^2$

3 In nuclear fission, the process

neutron ↘

$$n + {}^{235}U \rightarrow 3n + {}^{90}Kr + {}^{143}Ba$$

How much energy might be released during this reaction? If a nuclear warhead yields 10^{15} J of energy, how much uranium is required?

particle	mass (MeV/c^2) E
n	940
^{235}U	219000
^{90}Kr	84000
^{143}Ba	130000

$\Delta E = \Delta m \cdot c^2$

$940 + 219000 - 3 \cdot 940 - \cdot \cdot \cdot$

≈ -3000 MeV

In reality: get out 200 MeV

10^{15} J ~ 10^{28} MeV

~ 10^{26} U's

$m_U \sim 4 \times 10^{-25}$ kg

~ 40 kg.

4 In nuclear fusion, two particles come together to form a new particle: A + B → C. How is this possible if $m_A + m_B < m_C$? How fast does A need to move, if B is at rest, to create C, if $m_C = 3m_A = 3m_B$?

If B is at rest, how fast must A move?

Conserve E_{tot} & p_{tot}

$$(m_C c^2)^2 = E_C^2 - (c p_C)^2 = E_{tot}^2 - (c p_{tot})^2$$

$$\gamma = \left[1 - \frac{v^2}{c^2}\right]^{-1/2}$$

$$E_{tot} = m_B c^2 + \gamma m_A c^2$$

$$p_{tot} = \gamma m_A v$$

$$m_C^2 = m_A^2 + m_B^2 + 2\gamma m_A m_B$$

mass of C

$$\frac{v}{c} = \sqrt{1 - \left(\frac{2 m_A m_B}{m_C^2 - m_A^2 - m_B^2}\right)^2}$$