PHYS 2170

General Physics 3 for Majors

Fall 2021

Lecture 12

General relativity

September 20

1 Review Newtonian gravity.

$$F = \frac{G m_1 m_2}{r^2}$$

$$G \approx 6.67 \times 10^{-11}\ kg^{-1} m^3 s^{-2}$$

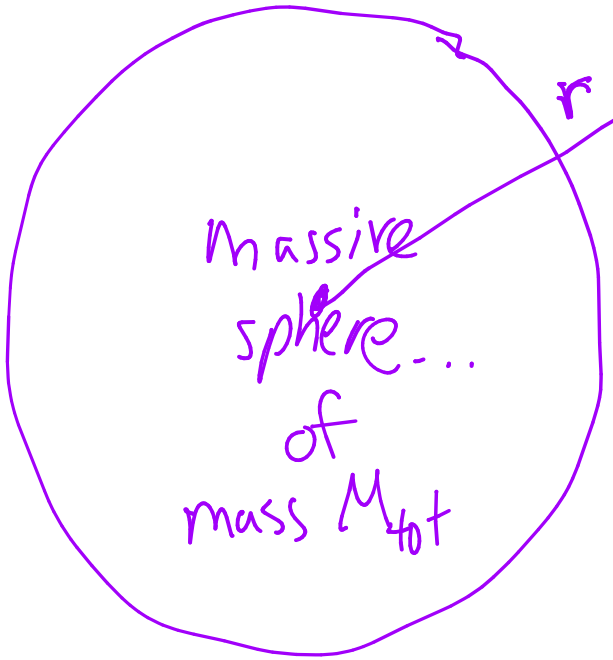
Analogous to Coulomb:

- all $m > 0$
- attractive

- analogue to electric potential:

$$\Phi_{\substack{\text{point} \\ \text{mass } M}} = -\frac{GM}{r}$$

$$\Phi = -\frac{GM_{tot}}{r}$$



- test mass m

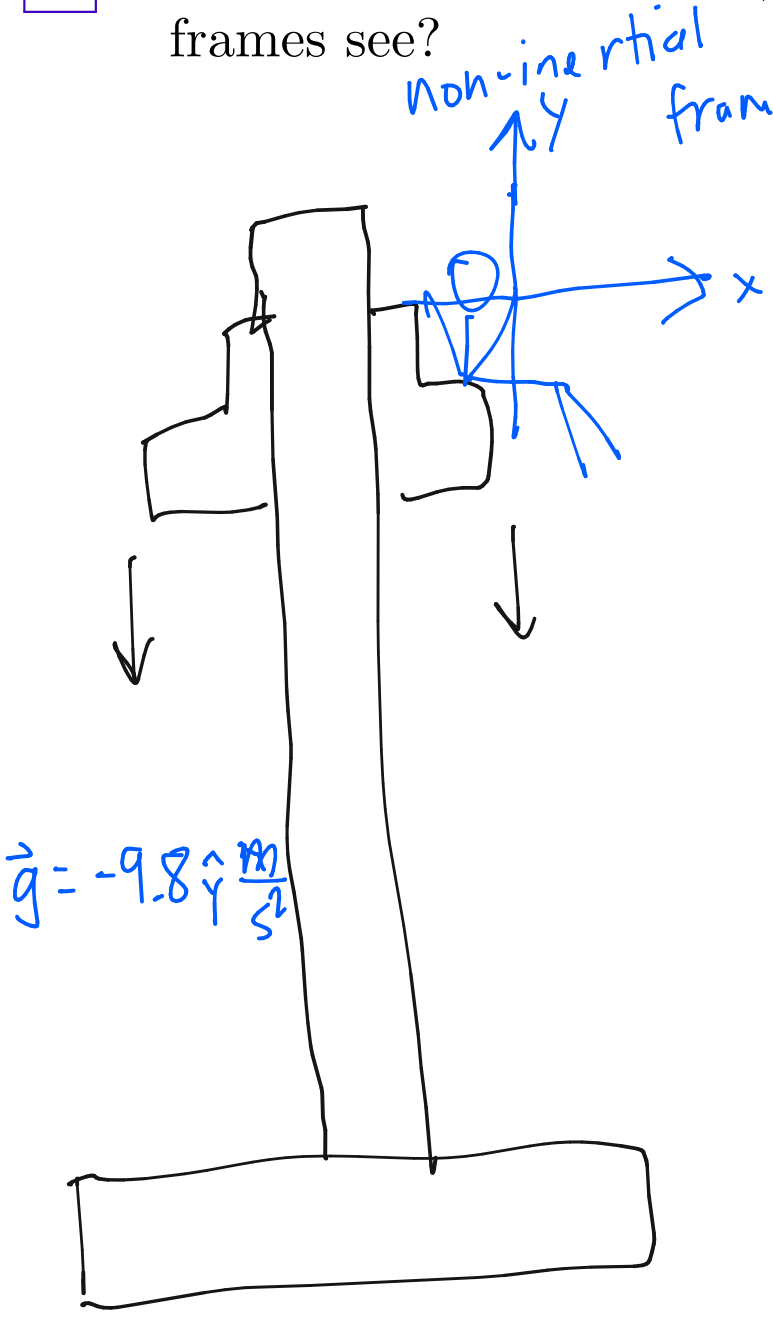
$$U_{grav} = m\Phi$$

electrost: $-\nabla V = \vec{E}$

here: $-\nabla \Phi = \vec{g}$

"grav pot"

2 In Newtonian mechanics, what do observers in non-inertial reference frames see?



$\vec{F}_{tot} = m(\vec{a} + \vec{g})$

$F_{grav} = m_{grav} g$

gravitational mass

$a \sim \frac{F}{m}$

inertial mass

EMPIRICAL "FACT": $m_{inert} = m_{grav}$.

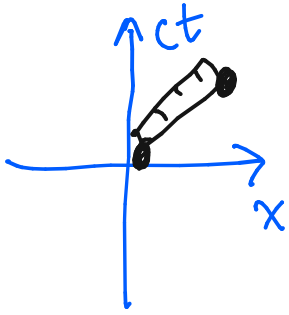
GR postulate:

no expt can tell diff btwn $\vec{a}$ and $\vec{g}$.

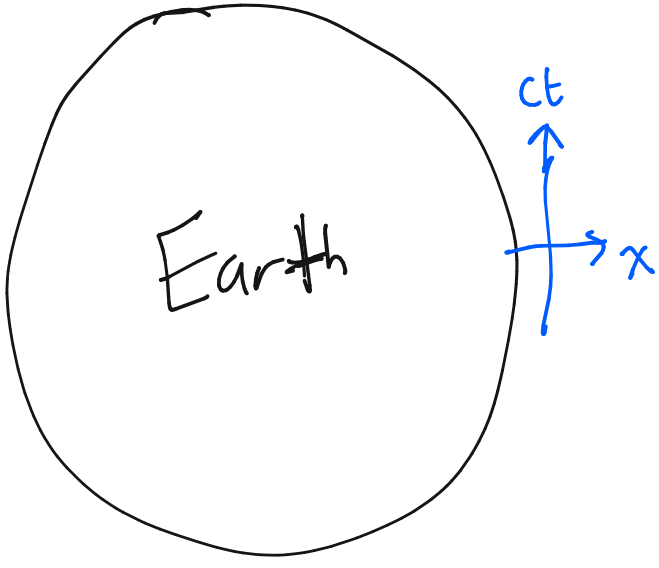
$\vec{F}_{non\text{-}grav,\ inertial} = m(\vec{a} + \vec{g})$

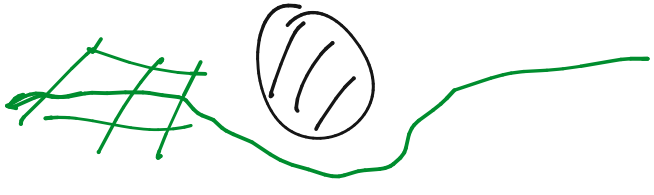
"Equivalence Principle".

3 What does it mean for spacetime to be curved?

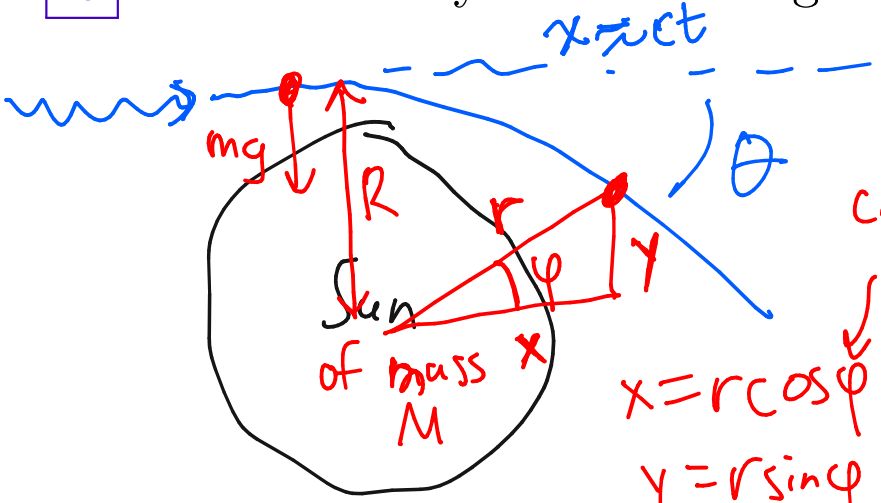


Δs^2





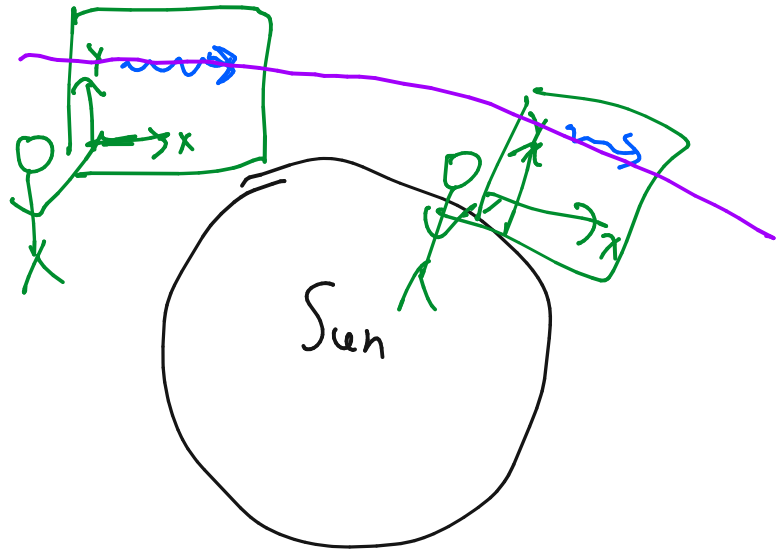
4 Estimate by how much light will bend as it moves around a sun.



$\cos\varphi \approx 1 - \frac{\varphi^2}{2}$

$x = r\cos\varphi \quad \sin\varphi \approx \varphi$

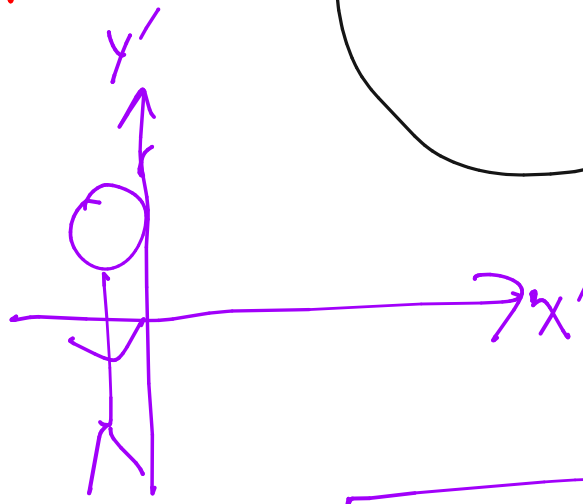
$y = r\sin\varphi \propto$



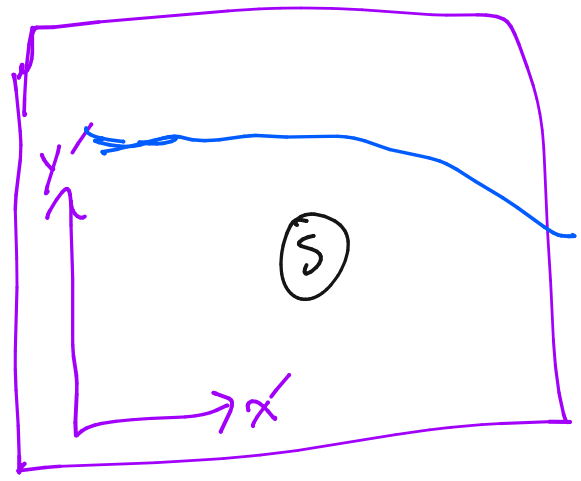
$$ma_y = -\frac{GMm}{r^2}\sin\varphi$$

$$a_y = -\frac{GM}{r^2}\sin\varphi = -\frac{GMy}{r^3}$$

$$r = (x^2+y^2)^{1/2} \Rightarrow ((ct)^2+y^2)^{1/2}$$

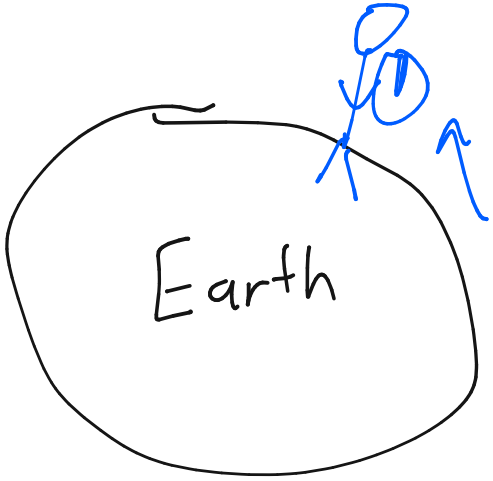


$$\Delta v_y = \int_{-\infty}^{\infty} dt\, a_y \sim -\frac{GM}{cR}$$

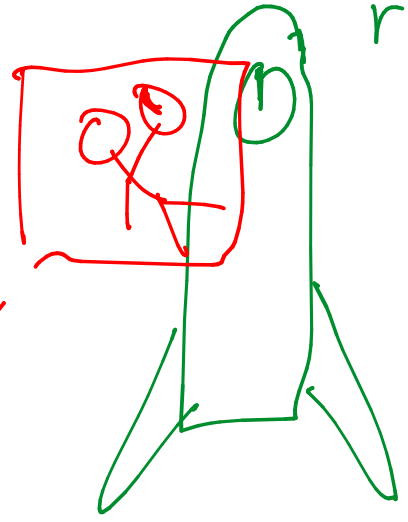


$$\theta \sim \frac{\Delta v_y}{v_x} \sim \frac{\Delta v_y}{c} \sim -\frac{GM}{c^2R}$$

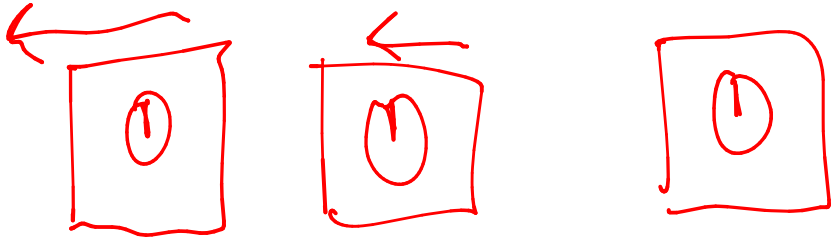
5 Estimate by how much time is dilated in the Earth's gravitational field.



tick "slowly" from grav. time dil.



rest $\Phi = 0$



right as it hit Earth

$v \ll c$

in Newtonian mechanics:

$$\frac{1}{2}mv^2 = -m\Phi$$

$$\Phi = -\frac{GM}{r}$$

$$v^2 = -2\Phi = \frac{2GM}{r}$$

$$t_{obs} \approx t_{clock}\,\gamma$$

$$\approx t_{clock}\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right)$$

$$t_{obs} = t_{clock}\left(1 + \frac{GM}{c^2 r}\right)$$