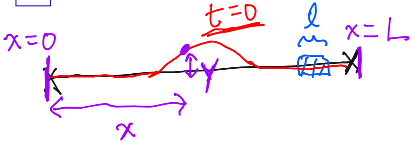
PHYS 2170
General Physics 3 for Majors
Fall 2021

Lecture 15
Waves on strings

September 27

1 Describe the transverse motion of a wave on a string in terms of $y(x,t)$.



tightly pulled string

- μ = mass per unit length
  - mass of a segment of length $\ell = \ell \cdot \mu$
- T = tension

GOAL: to find eqn which describes motion of string.

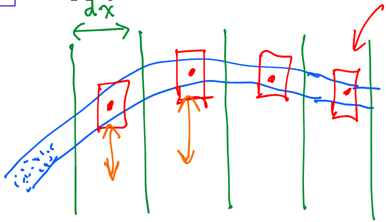
Initial ($t=0$) configuration of string characterized by $y(x)$

GOAL: find $y(x,t)$ = vertical displacement of string

x: horizontal dist. along string

t: time

$\boxed{2}$ Apply Newton's Second Law to little segments of the string.



$F = ma$ to "point masses"

- Mass of each segment $= \mu\, dx$

$F = ma$

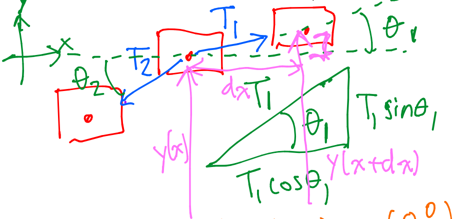
$F = (\underbrace{\mu dx}) a$

- explicit dx dependence
- expect "$F/a \sim dx$"
  (i.e. dx NOT in final eq.)

$a \approx$ "$\dfrac{d^2y}{dt^2}$"

Claim: most motion vertical

3 In a string with tension T, what is the net force on a segment lying between x and $x + \mathrm{d}x$?



net force on central point mass:

$$\vec{F} = \vec{T}_1 + \vec{T}_2$$

$$F_x = T_1 \cos\theta_1 - T_2 \cos\theta_2$$

$$F_y = T_1 \sin\theta_1 - T_2 \sin\theta_2$$

Assume $\theta_1, \theta_2 \ll 1$ ← much less than

[string weakly perturbed]

$$F_x \approx T_1\left(1 - \frac{\theta_1^2}{2}\right) - T_2\left(1 - \frac{\theta_2^2}{2}\right)$$

$$F_y \approx T_1 \theta_1 - T_2 \theta_2$$

At leading order in $\theta \ldots (\theta^0)$

$F_y \approx 0$, $F_x \approx T_1 - T_2$

$$\downarrow$$

$$T_1 = T_2 = T$$

At first order in θ,

$\underline{F_y \approx (\theta_1 - \theta_2) T}$ $\quad \Delta F_x \approx 0$

most motion is vertical

$$\tan\theta_1 = \frac{y(x+dx) - y(x)}{dx} \approx \theta_1$$

$$\theta_2 \approx \frac{y(x-dx) - y(x)}{dx}$$

4 What is a partial derivative?

Recap: $F = ma$

$$T(\theta_1 - \theta_2) = T\underbrace{\frac{y(x+dx)+y(x-dx)-2y(x)}{dx}} = \mu dx \; \boxed{\text{“}\frac{d^2y}{dt^2}\text{”}} = \frac{\partial^2 y}{\partial t^2}$$

acc. of fixed point mass $\left.\frac{d^2}{dt^2}y(x,t)\right|_{x \text{ fixed}}$

$y(x,t)$ is a function of 2 variables.

$$\frac{1}{dx}\left[y(x+dx,t) + y(x-dx,t) - 2y(x,t)\right]$$

↳ Taylor expand in x ...?

Define partial derivative:

$$\frac{\partial y}{\partial t} = \lim_{dt \to 0} \frac{y(x,t+dt) - y(x,t)}{dt}$$

$$\frac{\partial y}{\partial x} = \lim_{dx \to 0} \frac{y(x+dx,t) - y(x,t)}{dx}$$

5 Find the equation of motion for $y(x,t)$.

$$T\left(\frac{y(x+dx,t)+y(x-dx,t)-2y(x,t)}{dx}\right)=(\mu\, dx)\frac{\partial^2 y}{\partial t^2}$$

small parameter

Taylor expand $y(x+\overline{dx},t)=y(x,t)+\frac{\partial y(x,t)}{\partial x}\underbrace{dx}+\frac{1}{2}dx^2\cdot\frac{\partial^2 y}{\partial x^2}+\ldots$

$$\frac{1}{dx}\left[\cancel{y}+\cancel{\frac{\partial y}{\partial x}dx}+\frac{1}{2}dx^2\frac{\partial^2 y}{\partial x^2}+\cancel{y}-\cancel{\frac{\partial y}{\partial x}dx}+\frac{1}{2}(-dx)^2\frac{\partial^2 y}{\partial x^2}-\cancel{2y}\right]$$

$$\approx\frac{1}{dx}\cdot\frac{1}{2}\cdot 2\,dx^2\frac{\partial^2 y}{\partial x^2}=dx\frac{\partial^2 y}{\partial x^2}$$

$$T\cancel{dx}\frac{\partial^2 y}{\partial x^2}=\mu\cancel{dx}\frac{\partial^2 y}{\partial t^2}$$

no more dx!

$$\boxed{T\frac{\partial^2 y}{\partial x^2}=\mu\frac{\partial^2 y}{\partial t^2}}$$