PHYS 2170

General Physics 3 for Majors

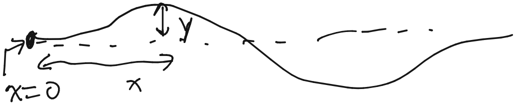
Fall 2021

Lecture 17

Reflection of waves

October 4

1 Review the wave equation on a string.



$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$$

wave speed $v = \sqrt{\frac{T}{\mu}}$

T = tension
μ = mass/unit length

Solution #1: plane waves ("building blocks")

$$y(x,t) = e^{i(kx - \omega t)} \qquad \omega = vk \text{ or } -vk$$

ordinary frequency (Hz) ↓

superimpose multiple: $\mathrm{Re}\left[e^{ik_1(x-vt)} + e^{ik_2(x-vt)} + \dots\right]$

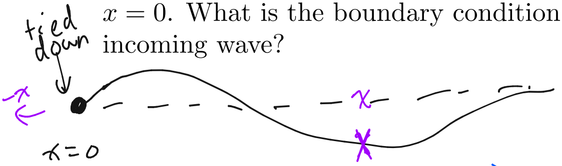
(s^{-1})

$2\pi f = \omega$ = angular frequency ; k = wave number

Solution #2: $y(x,t) = f(x-vt) + g(x+vt)$

$f(x-vt)$: right-moving; $g(x+vt)$: left-moving

2 Suppose that we tie the end of a string (that exists for $x \geq 0$) down at $x = 0$. What is the boundary condition on y? What happens to an incoming wave?



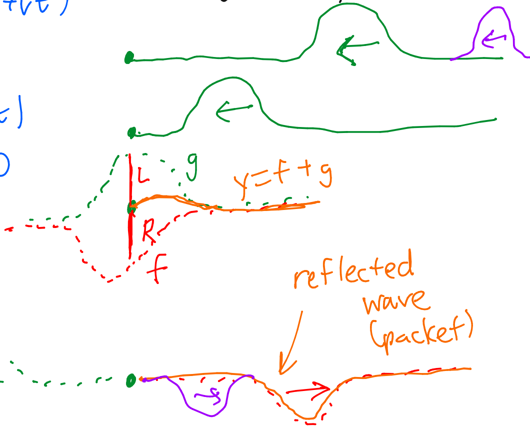
means that $y(x=0,t)=0$.
(boundary condition)

$y(x,t) = f(x-vt) + g(x+vt)$

Since $y(0,t)=0$:

$y(0,t) = f(0-vt) + g(0+vt)$
$\quad = f(-vt) + g(vt) = 0$

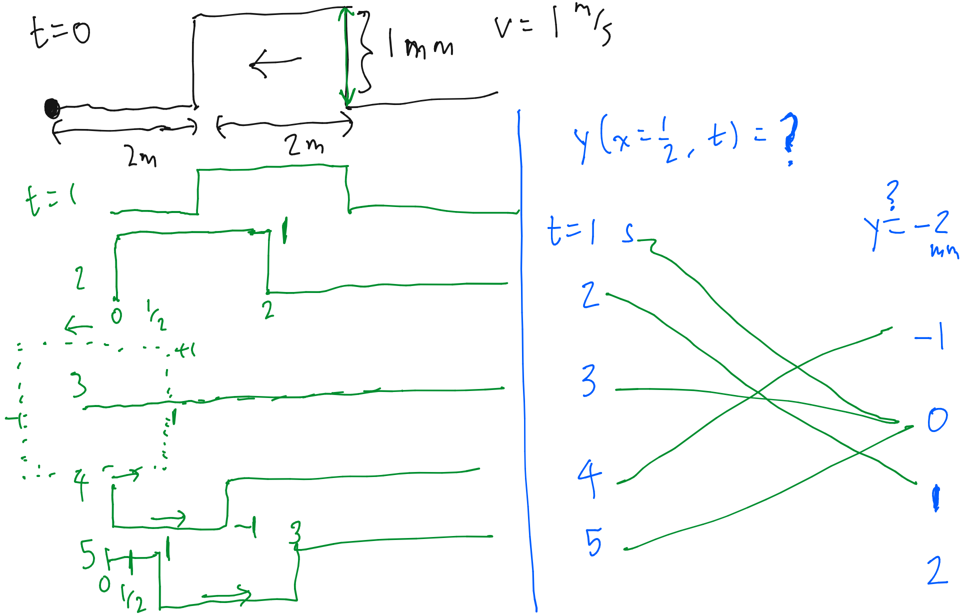
replace $g(vt) = -f(-vt)$
$\quad \to g(x) = -f(-x)$



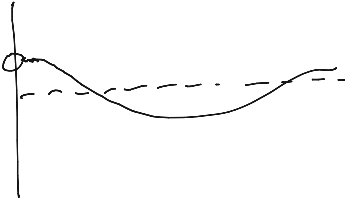
• destructive interference = $y=f+g$

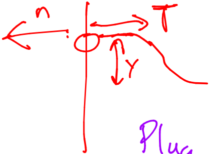
↑f ↓g sum $|f+g| < |f|$

3 Consider an incoming "square wave" on a string tied down at one end. Describe how the wave reflects at the boundary.



4 What are the boundary conditions on a string with a free end? How do waves reflect off of this?

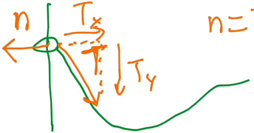




$$\left.\frac{\partial y}{\partial x}\right|_{x=0} = 0$$

- ring has negligible mass

$$F_{net} = m \cdot a = 0$$



$$n = T_x$$

$$F_x = F_y = 0$$

$$\Rightarrow \quad T_y = 0$$

Plug in: $f(x-vt) + g(x+vt)$

$$\frac{\partial y}{\partial x} = f'(x-vt) + g'(x+vt) = 0$$

$$g'(vt) = -f'(-vt) \quad \text{at } x=0$$

$$\int g'(vt) \cdot v\,dt = g(vt)$$

$$\int -f'(-vt) \cdot v\,dt = f(-vt)$$

$$g(vt) = f(-vt)$$

- constructive interference.