PHYS 2170

General Physics 3 for Majors

Fall 2021

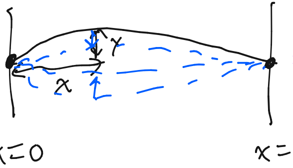
Lecture 19

Sound waves

October 8

1 Review how to solve the wave equation in a finite domain.

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$$



$\omega_n = \frac{n\pi v}{L}$

$n = 1, 2, 3, 4, \ldots$

Look for "simple" solns: $\underline{Y(x)e^{-i\omega t}} = y(x,t)$

– normal modes.

$\frac{\partial^2 y}{\partial t^2} = -\omega^2 y$

$= (-i\omega)^2 y$

$\frac{\partial^2 y}{\partial x^2} = e^{-i\omega t}\frac{d^2Y(x)}{dx^2}$; $\frac{d^2Y}{dx^2} = -\left(\frac{\omega}{v}\right)^2 Y$

Plug in: $Y(x) = e^{bx}$ ... find $b = i\omega/v$ or $-\frac{i\omega}{v}$

$Y(x) = A_1 e^{i\omega x/v} + A_2 e^{-i\omega x/v}$

Strategy: 1) use boundary condition at $x=0$
fix A_2 in terms of A_1

2) $x=L$ b/c: constrain ω (discrete val.)

$\boxed{2}$ What are the allowed frequencies if we have two free ends?

Boundary conditions at free end:

$$\left.\frac{\partial y}{\partial x}\right|_{x=0,L} = 0 \quad ; \quad Y'(0) = Y'(L) = 0$$

$$Y(x) = A_1 e^{i\omega x/v} + A_2 e^{-i\omega x/v}$$

$$Y'(0) = 0 = \frac{i\omega}{v}\left[A_1 \cancel{e^{i0}} - A_2 \cancel{e^{i0}}\right]$$

$$\Rightarrow \quad \underline{A_1 = A_2}$$

$$Y'(L) = 0$$

$$= \frac{i\omega}{v}\left[A_1 e^{i\omega L/v} - \cancel{A_2}^{A_1} e^{-i\omega L/v}\right]$$

$$0 = e^{i\omega L/v} - e^{-i\omega L/v}$$

$$0 = 2i \underbrace{\sin\frac{\omega L}{v}}_{=0}$$

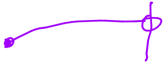
$$\boxed{\begin{aligned} \omega &= \frac{v}{L} n\pi \\ f &= \frac{\omega}{2\pi} = \frac{nv}{2L} \\ \lambda &= \frac{v}{f} \end{aligned}}$$

$$\underline{\omega = \frac{v}{L}\cdot n\pi} \qquad n = 0, 1, 2, \ldots,$$

2 Fixed:

$$\omega_n = \frac{n\pi v}{L}$$

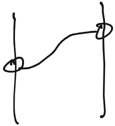
one free:

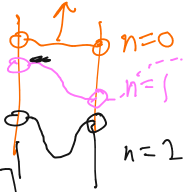


$$\omega_\ell = \frac{\pi v}{L}\ell$$

$$\ell = 1, 3, 5, \ldots$$

2 free:



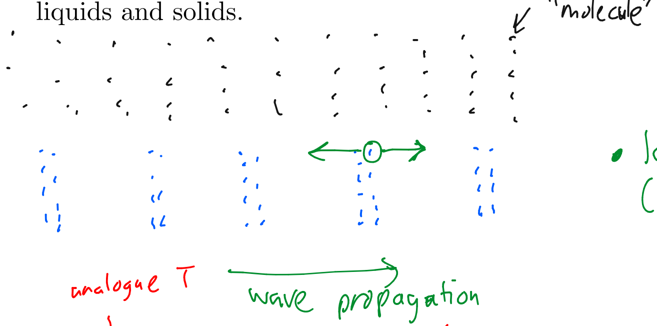


Wavelength for $n \neq 0$ modes:

$$\lambda_1 = 2L$$

$$\boxed{\lambda_n = 2L/n}$$

3 Give a formula for the speed of sound, and estimate it in typical gases, liquids and solids.



- longitudinal wave (sound in liquid/gas some solids)

analogue T

$$v = \sqrt{\frac{Y}{\rho}}$$

analogue of μ

Y = Young's modulus ["spring constant"] (how squishy)

$v \sim 3000$ m/s

$Y \sim 10^9 - 10^{11}$ Pa $\sim$ N/m^2

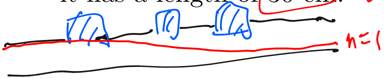
$v_{liquid} \sim 1000$ m/s

ρ = mass density = $\frac{\text{mass}}{\text{volume}}$

$v_{air} \sim 340$ m/s

$\rho \sim 4000$ kg/m^3

4 A small woodwind instrument can be approximated by a tube with two "free" ends. Estimate the fundamental frequency of this instrument if it has a length of 30 cm. $= L$



Q: „Sound waves resonate.
– "free" boundary conditions.

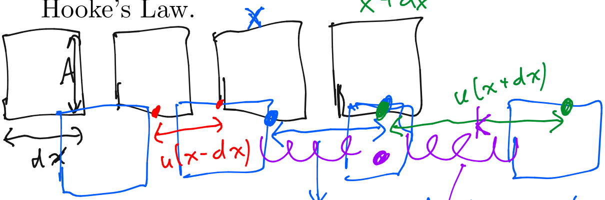
• Wavelength of fundamental mode:

$$\lambda_n = \frac{1}{n} \cdot 2 \cdot L = 60\ \text{cm} \cdot \frac{1}{n}$$

frequency (dominated by $n=1$): $f = \frac{v}{\lambda} = \frac{v}{2L} = \frac{340\ \frac{m}{s}}{2 \cdot 30\ \text{cm}}$

$\sim 500\ \text{Hz}$.

5 Describe the stresses in a solid and derive a continuum version of Hooke's Law.



$u(x)$ = displacement of x

this spring is stretched by $u(x+dx) - u(x)$

$$F_{net} = k\left[u(x+dx) - u(x)\right] - k\left[u(x-dx) - u(x)\right]$$

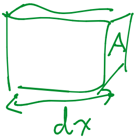
$$= k\left[u(x+dx) + u(x-dx) - 2u(x)\right] \approx k \cdot dx^2 \frac{\partial^2 u}{\partial x^2}$$

Young's modulus

$$k = \frac{A}{dx} \cdot Y$$

elastic solid

6 Derive the wave equation in a solid.



$$F_{net} = m \frac{\partial^2 u}{\partial t^2}$$

$$m = \rho \cdot A \cdot dx = \rho \times vol$$

$$K\, dx^2 \frac{\partial^2 u}{\partial x^2} = A Y\, dx \frac{\partial^2 u}{\partial x^2}$$

$$\rho \cancel{A} \cancel{dx} \frac{\partial^2 u}{\partial t^2} = \cancel{A} Y \cancel{dx} \frac{\partial^2 u}{\partial x^2}$$

$$\cancel{\rho} \frac{\partial^2 u}{\partial t^2} = \frac{Y}{\rho} \frac{\partial^2 u}{\partial x^2}$$

$$v^2 = \frac{Y}{\rho} \qquad v = \sqrt{\frac{Y}{\rho}}$$