PHYS 2170

General Physics 3 for Majors

Fall 2021

Lecture 20

Waves in solid rods

October 11

1 If you bang a metal spoon of length 20 cm against a table, is it audible?

$$V = \sqrt{\frac{Y}{\rho}}$$ ← Young's modulus, ← mass density

yes!

$v \sim 5000$ m/s

lowest possible frequency of sound?

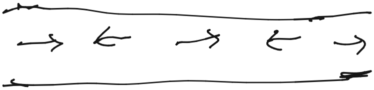
first: longest possible wavelength of sound waves is

$$\lambda = 40 \text{ cm} \qquad L = 20 \text{ cm}$$
$$= \frac{2L}{n} \quad (n=1)$$

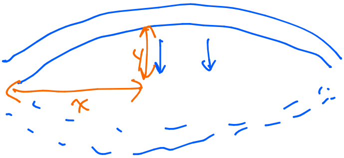
So lowest possible vibration frequency is:

$$f = \frac{v}{\lambda} \approx 12.5 \text{ kHz.} \longrightarrow \text{not what we hear}$$

2 Describe qualitatively why a solid rod would resist being bent (or "flexed") in the transverse direction.



Sound wave



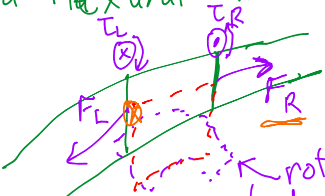
flexural wave ✓

(much lower frequency)

$$\boxed{\frac{\partial^2 y}{\partial t^2} = -K\frac{\partial^4 y}{\partial x^4} \quad (K>0)}$$

dispersive b/c not $\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$

In a flexural wave



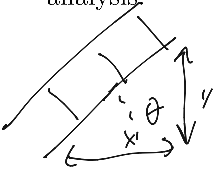
rotation not dominant

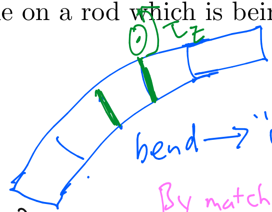
$\tau_R = \vec{\tau}_L + d\vec{\tau}$

$$\tau_{net} = |\tau_R| - |\tau_L| + d\vec{r} \times \vec{F}_R = 0$$

$$\boxed{d\tau_z = -dx\, F_y}$$

3 Estimate the torque on a rod which is being flexed by dimensional analysis.





both rods have $\frac{\partial y}{\partial x} \neq 0$

bend → "out of equilibrium"

Claim: ???

$$\tau_z = C \frac{\partial^2 y}{\partial x^2}$$

By matching units:

$$\boxed{C = Y w^4} \cdot \text{const.}$$

$\ldots, 3, \frac{1}{8\pi}, 8\pi$

C might depend on:

Young's modulus $[Y] = J/m^3 = \frac{kg}{m \cdot s^2} = \frac{N}{m^2}$

mass density $[\rho] = kg/m^3$

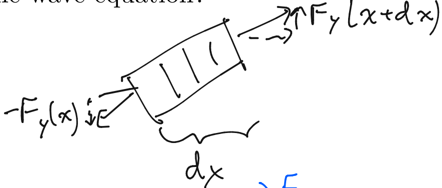
moment of inertia / or width $[w] = m$

What are SI units? (i.e. dimensions)

$$[C] = \frac{[\tau_z]}{\left[\frac{\partial^2 y}{\partial x^2}\right]} = \frac{N \cdot m}{\frac{[y]}{[x]^2}} = \frac{N \cdot m}{\frac{1}{m}} = N \cdot m^2$$

4 Derive the equation governing “flexural” (transverse) dynamics of the rod. Is it equivalent to the wave equation?

$$\underbrace{m}\frac{\partial^2 y}{\partial t^2} = \underbrace{F_{net,y}}$$



$\rho A\, dx$

$\rho w^2\, dx$

$$F_y(x+dx) - F_y(x) \approx \frac{\partial F_y}{\partial x}\,dx$$

$$\approx -\frac{\partial^2 \tau_z}{\partial x^2}\,dx$$

$$\approx -Y w^4 \frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 y}{\partial x^2}\right)dx$$

$$\rho \cancel{w^2}\,\cancel{dx}\,\frac{\partial^2 y}{\partial t^2} = -Y w^{\cancel{4}\,2}\,\cancel{dx}\,\frac{\partial^4 y}{\partial x^4}$$

$$\boxed{\frac{\partial^2 y}{\partial t^2} = -\frac{Y}{\rho} w^2 \frac{\partial^4 y}{\partial x^4}}$$

constant ≈ 1

NOT usual wave equation!

5 Estimate the fundamental frequency at which a rod oscillates due to flexural modes. Return to the question at the start of the lecture.

$$\frac{\partial^2 y}{\partial t^2} = -\frac{Y w^2}{\rho}\frac{\partial^4 y}{\partial x^4}$$

$$y = e^{ikx - i\omega t}$$

$$(-i\omega)^2 y = -\frac{Y w^2}{\rho}(ik)^4 y$$

$$\omega^2 = \frac{Y w^2}{\rho} k^4$$

$$\omega = \pm v w k^2$$

Speed of sound

Like before, flex mode of longest $\lambda \approx 2L$ (not exact)

$$k = \frac{2\pi}{\lambda} \approx \pi/L$$

(Hz)

$$f_{flex} = \frac{1}{2\pi}\omega = \frac{v w}{2\pi}\left(\frac{\pi}{L}\right)^2$$

$$\approx f_{sound} \cdot \frac{\pi w}{L}$$

12.5 kHz

$w \approx \frac{1}{2}$ cm

$$f_{flex} \sim 12\text{ kHz}\ \frac{3 \cdot 1/2}{20} \sim 900\text{ Hz}$$