PHYS 2170

General Physics 3 for Majors

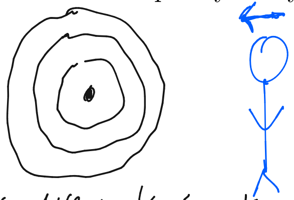
Fall 2021

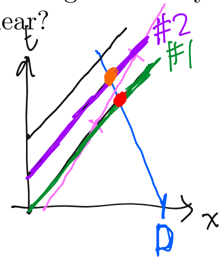
Lecture 22

Non-relativistic Doppler effect

October 15

1 Consider a stationary source of (sound) waves, emitting waves at frequency f. If you are traveling at velocity v_o relative to the source, what frequency will you hear?





pulse every $1/f$ seconds.

$x_0(t=0) = D$

$x_0(t) = D - v_0 t$

speed of sound v:

$x_1(t) = vt$

$x_2(t) = v\left(t - \frac{1}{f}\right)$

At what time t_1 does O hear pulse 1?

$$\boxed{t_1 = \frac{D}{v+v_0}}$$

$x_0 = D - v_0 t_1 = v t_1$

$t_2 - t_1 = \frac{1}{f} \frac{v}{v+v_0}$

$\frac{1}{t_2 - t_1} = \boxed{f_0 = f\,\frac{v_0 + v}{v}}$

time t_2 to hear pulse 2:

$D - v_0 t_2 = v\left(t_2 - \frac{1}{f}\right)$

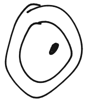
$D + \frac{v}{f} = (v_0 + v) t_2$

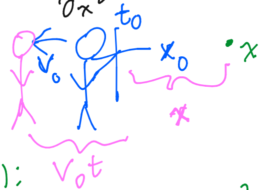
$$t_2 = \overbrace{\frac{D}{v+v_0}}^{t_1} + \frac{1}{f}\,\frac{v}{v+v_0}$$

$\boxed{2}$ Re-derive this result by thinking about the wave equation in a (non-relativistic) moving frame of reference.

molecular displacement

In a stationary frame (speaker's): $\dfrac{\partial^2 u}{\partial t^2} = v^2 \dfrac{\partial^2 u}{\partial x^2}$ (for sound)





$$\boxed{\begin{aligned} t &\approx t_0 \\ x_0 &\approx x + v_0 t \end{aligned}} + \cancel{\text{const.}}$$

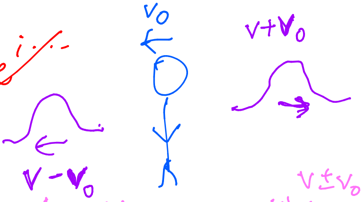
transform to (x_0, t_0):

$$\left(\frac{\partial}{\partial t_0} + v_0 \frac{\partial}{\partial x_0}\right)^2 u = v^2 \frac{\partial^2 u}{\partial x_0^2}$$

Plug in $e^{ik_0 x_0 - i\omega_0 t_0}$

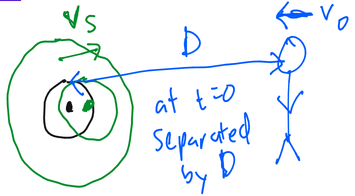
$$(-i\omega_0 + ik_0 v_0)^2 \cancel{e^{i\cdots}} = v^2 (ik_0)^2 \cancel{e^{i\cdots}}$$
$$= (ik_0 v)^2$$

$$-i\omega_0 + ik_0 v_0 = \pm i k_0 v$$
$$\omega_0 = k_0 (v \pm v_0)$$



$k_0 = k = \dfrac{\omega}{v}$: $\omega_0 = \omega \dfrac{v \pm v_0}{v}$

3 What is the Doppler effect if the source is moving?



Again: observer's position

$$x_o(t) = D - v_o t$$

pulse 1:

$$x_1(t) = vt$$

pulse 2:

$$x_2(t) = v\left(t - \frac{1}{f}\right) + \frac{v_s}{f}$$

O hears 1 when:

$$D - v_o t_1 = v t_1$$

$$t_1 = \frac{D}{v + v_o}$$

Hear pulse 2:

$$D - v_o t_2 = v\left[t_2 - 1/f\right] + \frac{v_s}{f}$$

$$t_2 = \frac{D}{v + v_o} + \frac{1}{f}\frac{[v - v_s]}{v + v_o}$$

Time between pulses:

$$t_2 - t_1 = \frac{1}{f_o} = \frac{1}{f}\,\frac{v - v_s}{v + v_o}$$

obs. frequency

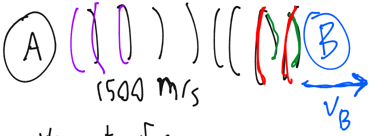
sound speed

ob speed

$$\boxed{f_o = f\,\frac{v + v_o}{v - v_s}}$$

source speed

4 A stationary submarine A emits sound waves (sonar) and attempts to hear reflected sound moving off of other submarines. If hostile submarine B moves at 1 m/s, how accurately must A detect frequency to "hear" B in motion? (Speed of sound in water is about 1500 m/s.)



emit at f_A

Step 1: B moving away from A

$$f_{B,obs} = f_A \frac{1500-1}{1500}$$

Step 2: Reflected sound waves?

as viewed by B:

$f_{B,obs}$ again

Step 3: signal come from objects moving together

$$f_{A,rec} = f_{B,obs} \frac{v + v_0}{v - v_s}$$

$$= f_{B,obs} \frac{v+0}{v-(-v_B)}$$

$$= f_{B,obs} \frac{1500}{1501}$$

$$f_{A,rec} = f_A \underbrace{\frac{1499}{1500} \frac{1500}{1501}}_{=1499/1501}$$