PHYS 2170
General Physics 3 for Majors
Fall 2021

Lecture 23

Light and the relativistic Doppler effect

October 18

1 Review Maxwell's equations.

Fundamental equations of E&M: Maxwell's Equations

$$\Phi_E = \oint \vec{E}\cdot d\vec{a} = q_{enc}/\varepsilon_0 = 0$$

$$\Phi_B = \oint \vec{B}\cdot d\vec{a} = 0.$$

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\Phi_B$$

$$\oint \vec{B}\cdot d\vec{l} = \cancel{\mu_0 I_{enc}} + \mu_0\varepsilon_0 \frac{d\Phi_E}{dt}$$

divergence thm:

Stoke's Thm:

$$\boxed{\begin{gathered}\nabla\cdot\vec{E}=0\\ \nabla\cdot\vec{B}=0\end{gathered}}$$

$$\nabla\times\vec{B} = \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

["free space": no ~~free point~~ free charges/currents]

"Plane wave" solutions

wave moving in x-direction

$$\vec{E} = \underbrace{e^{-i\omega t+ikx}}\vec{E}_0$$

const.

$$\vec{B} = e^{-i\omega t+ikx}\vec{B}_0$$

$$\nabla\cdot\vec{E}=0$$

$$\frac{\partial E_x}{\partial x}=0$$

$$ik\underbrace{E_{0,x}e^{-i\cdots}}$$

$$ik\,E_{0,x}=0$$

$$E_{0,x}=0$$

similarly: $B_{0,x}=0$

2 Deduce the existence of electromagnetic waves (i.e. light).

$c = 3\times10^8$ m/s
speed of light

This derivation holds in "vacuum"

$E_{0,y} \neq 0 \qquad B_{0,z} \neq 0$

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$$

$$ik E_y = i\omega B_z$$

$$-\frac{\partial B_z}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t}$$

$$-ik B_z = -i\omega \mu_0 \varepsilon_0 E_y$$

$$\frac{E_y}{B_z} = \frac{i\omega}{ik} = \frac{\omega}{k}$$

$$\frac{E_y}{B_z} = \frac{k}{\omega}\cdot\frac{1}{\mu_0\varepsilon_0}$$

$$\frac{\omega}{k} = \frac{k}{\omega}\cdot\frac{1}{\mu_0\varepsilon_0}$$

$$\left(\frac{\omega}{k}\right)^2 = \frac{1}{\mu_0\varepsilon_0} = c^2$$

$$\omega = \pm ck$$

In material:

$$\boxed{\begin{array}{l}\mu_0 \to \mu \\ \varepsilon_0 \to \varepsilon\end{array}}$$

$$\omega = \pm\frac{c}{n}k$$

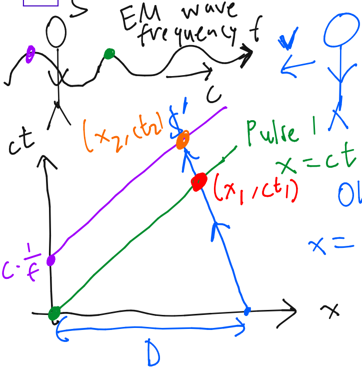
$$n = \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}}$$

(index of refraction)

3 Discuss briefly the frequency/wavelength ranges of various electromagnetic waves.

type	λ	$f = \frac{c}{\lambda}$ (c: 3×10^8 m/s)
X-rays	~ 0.1 nm	3×10^{18} Hz
ultraviolet		
blue light	450 nm	$\sim 8\text{-}9\times10^{14}$ Hz
red light	700 nm	$\sim 5\times10^{14}$ Hz
infrared...		
microwave	1 cm	~ 30 GHz
radio	1 m	3×10^8 Hz $\sim$ 0.3 GHz

4 Derive the relativistic Doppler effect.



Pulse 2: $x = c\left(t - \frac{1}{f}\right)$

Observer S': $x = D - vt$

S' sees #1:

$x_1 = D - vt_1 = ct_1$

$t_1 = \frac{D}{c+v}$ $\quad x_1 = \frac{cD}{c+v}$

S' sees #2:

$x_2 = D - vt_2 = c\left(t_2 - \frac{1}{f}\right)$

$D + \frac{c}{f} = (c+v)t_2$

$t_2 = t_1 + \frac{1}{f}\frac{c}{c+v}$

$x_2 = x_1 - \frac{1}{f}\frac{vc}{c+v}$

in frame S

$\Delta t = t_2 - t_1$

But S': $\Delta t' = \frac{\Delta t}{\gamma}$

(moving clock runs slow)

$\gamma = \sqrt{\frac{1}{1 - v^2/c^2}}$

$\Delta t' = \sqrt{1 - v^2/c^2}\left(\frac{1}{f}\frac{1}{1+\frac{v}{c}}\right)$

$= \sqrt{\left(1-\frac{v}{c}\right)\left(1+\frac{v}{c}\right)}\,\frac{1}{f}\frac{1}{1+v/c} = \sqrt{\frac{1-v/c}{1+v/c}}\,\frac{1}{f}$

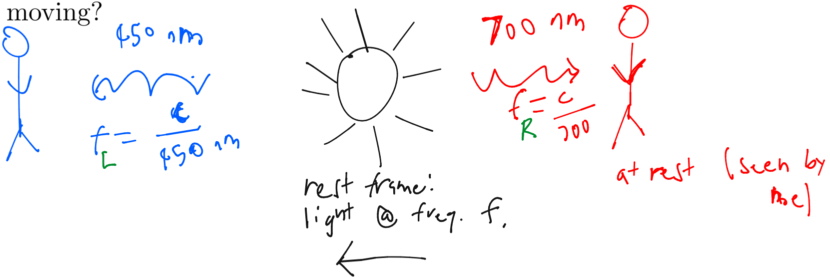
observed in frame S'

$f' = \frac{1}{\Delta t'} = f\sqrt{\frac{1+v/c}{1-v/c}}$ source moving toward obs.

if source moves away from obs:

$f' = f\sqrt{\frac{1-v/c}{1+v/c}}$

5 An object emits light of a fixed frequency. If it appears red to an observer standing on one side and blue on the other, how fast is it moving?



$$\frac{f_L = \frac{c}{450} = f\sqrt{\frac{c+v}{c-v}}}{f_R = \frac{c}{700} = f\sqrt{\frac{c-v}{c+v}}}$$

$$\frac{\frac{c}{450}}{\frac{c}{700}} = \frac{70}{45} = \frac{14}{9} = \frac{\cancel{f}\sqrt{\frac{c+v}{c-v}}}{\cancel{f}\sqrt{\frac{c-v}{c+v}}} = \frac{c+v}{c-v}$$

$$14(c-v) = 9(c+v)$$

$$\boxed{v = \frac{5}{23}c}$$