

PHYS 2170
General Physics 3 for Majors
Fall 2021

Lecture 24

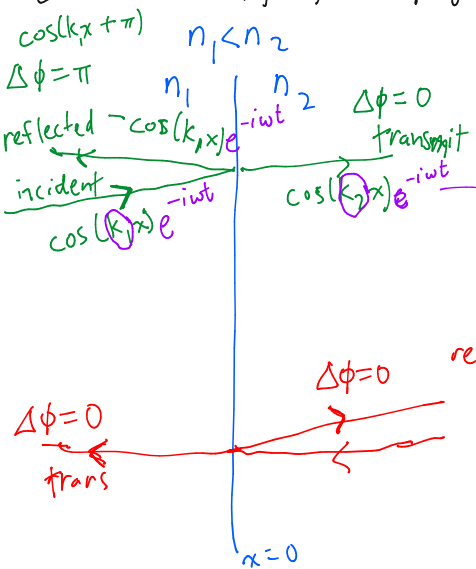
Interference of light in thin films and between two slits

October 20

- 1 Discuss the boundary conditions for how light reflects at an interface between two different materials.

EM waves ("light") propagates at velocity c/n

- if no material: $n=1$
- in a material: $n \geq 1$



$$k_1 = \frac{2\pi}{\lambda_1} \quad k_2 = \frac{2\pi}{\lambda_2}$$

incoming frequency f_1

$$f_2 = f_1$$

- frequency, not wavelength, is constant across interface.

2

Materials A and B have index of refraction $1 < n_A = 2 < n_B = 3$. If a thin layer of A of width w is painted on a thick slab of B, what is the longest wavelength for which there is constructive interference due to the film of A? What about destructive interference?



Wave has wavelength λ_0 in air.

$$\frac{c}{n_A} = f \lambda_A \quad \frac{c}{1} = f \lambda_0$$

$$\Delta\phi = \phi_{\text{path 2}} - \phi_{\text{path 1}} = 2\phi_X = \frac{4\pi w}{\lambda_A}$$

$$= \frac{8\pi w}{\lambda_0}$$



constructive interference:

$$\Delta\phi = 2\pi, 4\pi, \dots$$

destructive interference?

$$\Delta\phi = \pi, 3\pi, \dots$$

constructive:

destructive:

path 1: $\phi_{\text{bef}} + \pi + \phi_{\text{aft}}$
 (trans) (A/B refl)

path 2: $\phi_{\text{bef}} + 0 + \phi_X + \pi$

$\lambda_A = \frac{\lambda_0}{n_A} + \phi_Y + 0 + \phi_{\text{after}}$
 (trans)

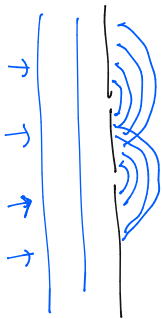
$$\phi_X = \phi_Y = \frac{w}{\lambda_A} 2\pi$$

$$\frac{1}{4} \lambda_0 = w$$

$$\frac{1}{8} \lambda_0 = w$$

3

Describe how light (and waves in general) propagate away from a coherent point source.

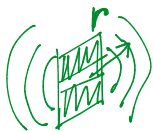


1d:

$x < 0$ $x > 0$
 $E_y \sim e^{ik(x+ct)}$ $E_y \sim e^{ik(x-ct)}$
 proportional

$$k = \frac{2\pi}{\lambda}$$

2d:



$$E \sim \frac{1}{\sqrt{r}} e^{ik(r-ct)}$$

$$I \sim \frac{\text{energy}}{\text{volume}} \sim \frac{1}{2} \epsilon_0 |E|^2$$

Wavefront energy $\sim \pi r \times I \sim r$ -ind.

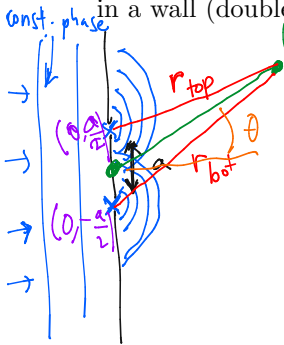
3d:



$$E \sim \frac{1}{r} e^{ik(r-ct)}$$

4

Describe the amplitude of light which passes through a pair of thin slits in a wall (double-slit interference).



- const phase in incoming wave
- at openings slit, light is in phase

$$(x, y) = (r \cos \theta, r \sin \theta)$$

$$E(x, y) \approx \frac{1}{\sqrt{r_{top}}} e^{ik(r_{top} - ct)} + \frac{1}{\sqrt{r_{bot}}} e^{ik(r_{bot} - ct)}$$

$$\begin{aligned} r_{top} &= \sqrt{x^2 + (y - \frac{a}{2})^2} & r \gg a \\ &= \sqrt{r^2 - 2\frac{a}{2}r \sin \theta + \frac{a^2}{4}} \\ &\approx r \sqrt{1 - \frac{a}{r} \sin \theta} \approx r - \frac{a}{2} \sin \theta \end{aligned}$$

$$r_{bot} \approx r + \frac{a}{2} \sin \theta$$

$$E \approx \frac{1}{\sqrt{r}} e^{-ikct} \left(e^{ik(r - \frac{a}{2} \sin \theta)} + e^{ik(r + \frac{a}{2} \sin \theta)} \right)$$

$$e^{ikr} 2 \cos\left(\frac{ka}{2} \sin \theta\right)$$

5

Describe the intensity of light as viewed on a screen far from the original slits.