PHYS 2170

General Physics 3 for Majors

Fall 2021

Lecture 27

Quantum wave functions

October 27

1 Review the notion of a photon. State wave-particle duality.

photon = particle of light (quantum of E&M)

energy $\boxed{\begin{aligned} E &= \hbar\omega \\ &= hf \end{aligned}}$ $\omega = 2\pi f$

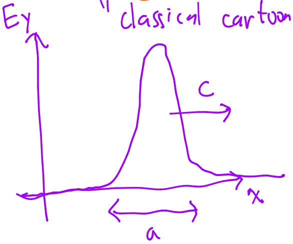
momentum $\boxed{\begin{aligned} p &= \hbar k \\ &= h/\lambda \end{aligned}}$ $k = \frac{2\pi}{\lambda}$

h = Planck's const
$= 6.6 \times 10^{-34}$ J·s

$\hbar = \frac{1}{2\pi} h$

"Particle-wave duality": photon can be thought of as "both wave and particle"

"classical cartoon"



In QM, all particles obey
$E = \hbar\omega$ $p = \hbar k$

2 If wave-particle duality applies to all matter, then what is the wavelength of an (electron, proton, baseball) moving at 10^6 m/s?

baseball:

$m \approx 0.1$ kg (100 g)

$v \approx 10^6 \frac{m}{s}$

$p = mv \approx 10^5 \frac{kg \cdot m}{s}$

$$\lambda = \frac{h}{p} = \frac{6.6 \times 10^{-34}}{10^5}$$

$= 6.6 \times 10^{-39}$ m

proton:

$m \approx 1.7 \times 10^{-27}$ kg

$$\lambda_{bb} = \frac{h}{m_{bb} v}$$

$$\lambda_p = \frac{h}{m_p v}$$

$$= \frac{m_{bb}}{m_p} \lambda_{bb}$$

$$\sim \frac{10^{26}}{1.7} (6.6 \times 10^{-39} \text{ m})$$

$(3-4 \times 10^{-13})$

$\rightarrow$ 0.3-0.4 pm

electron:

$m \approx 10^{-30}$ kg

$$\lambda_e = \lambda_p \underbrace{\frac{m_p}{m_e}}_{2000}$$

$\lambda_e \sim 6 \times 10^{-10}$ m

~ 0.6 nm

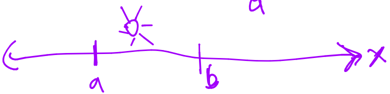
3 Describe the quantum wave function of a particle.

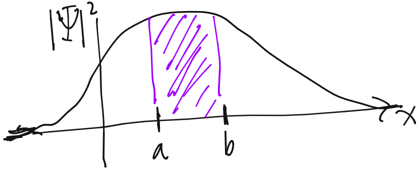
E&M: wave is made out of $\vec{E}$ & $\vec{B}$

- if wave angular freq. ω ... photons each has $E = \hbar\omega$
- intensity of light $= \frac{\text{energy}}{\text{volume}} \sim \epsilon_0 |\vec{E}|^2$
- $\frac{\text{intensity}}{\hbar\omega} \sim \frac{\#\text{ of photons}}{\text{volume}} \sim \frac{\epsilon_0}{\hbar\omega} \underbrace{|\vec{E}|}_{\text{wave amplitude.}}{}^2$

The quantum wave function of one particle (e^-) is $\Psi(x,t)$, where $|\Psi|^2 \sim$ probability of finding a particle near point x.

$$\mathbb{P}(a \le x \le b) = \int_a^b dx\, |\Psi|^2$$





4 What is the normalization condition? Is a plane wave normalized?

For one particle: $\int_{-\infty}^{\infty} dx\, |\Psi|^2 = 1$.

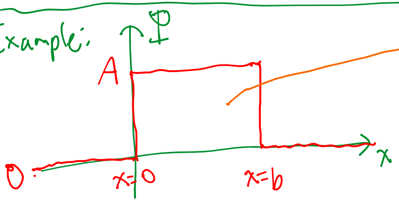
$\mathbb{P}(-\infty < x < \infty)$

$= \mathbb{P}(\text{seeing particle somewhere})$

$= 1$

"normalization condition"

Example:



$$\int_{-\infty}^{\infty} dx\, |\Psi|^2 = \int_0^b dx\, |\Psi|^2$$

$$= \int_0^b dx\, |A|^2 = b \cdot |A|^2$$

$= 1$, so $A = \frac{1}{\sqrt{b}}$

$$\Psi = \sum \Psi_k e^{ikx - i\omega t}$$

Guess: $\Psi = e^{ikx - i\omega t} \cdot A$

Is this "plane" wave associated to one particle?

$$\int_{-\infty}^{\infty} dx\, |\Psi|^2 = \int_{-\infty}^{\infty} dx\, |A|^2 = \infty \cdot |A|^2 = \infty$$

one plane wave is not normalized.

5 Is it possible to see the diffraction of electrons/neutrons through a crystal lattice?