PHYS 2170

General Physics 3 for Majors

Fall 2021

Lecture 28

Measurement in quantum mechanics

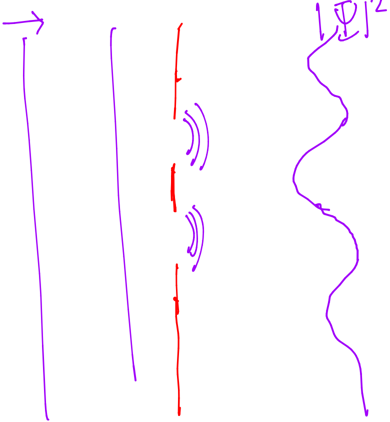
October 27

1 Review the wave function of a particle. Discuss the "two-slit" experiment.

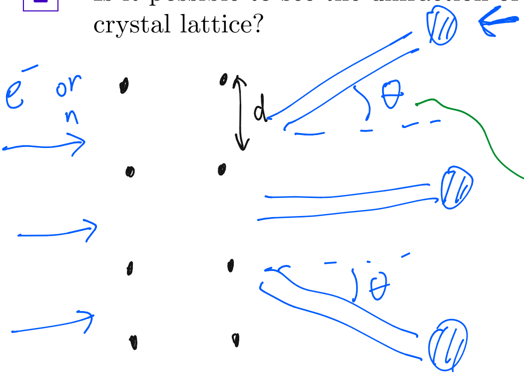
$\Psi(x,t)$ = complex-valued (one dimension) : $\int_a^b dx\, |\Psi|^2 = \mathbb{P}(\text{particle in } a \le x \le b)$

$|\Psi|^2 = (\mathrm{Re}\,\Psi)^2 + (\mathrm{Im}\,\Psi)^2$

- Wave function normalization: $\int_{-\infty}^{\infty} dx\, |\Psi|^2 = 1.$ (particle is somewhere)



2 Is it possible to see the diffraction of electrons/neutrons through a crystal lattice?



We'll see multiple peaks in $|\Psi|^2$ (# of particles collected)

$$\sin(\theta) = \frac{\lambda}{d} \qquad \lambda \sim d$$

For matter wave:

$$\lambda = \frac{h}{p}$$

Crystal: $d \sim 0.1$ nm

Particle (e^- & n) energy of 10^{-18} J (6 eV)

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$p = \sqrt{2mK} \qquad \lambda = \frac{h}{\sqrt{2mK}}$$

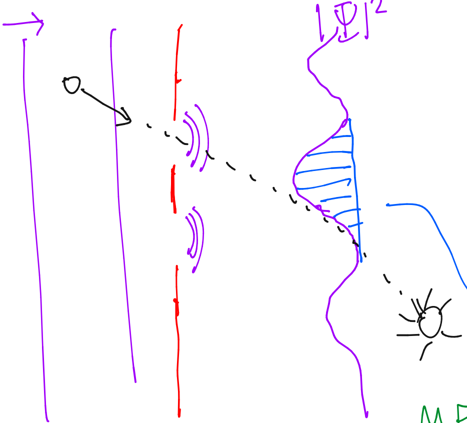
electron: $m \sim 10^{-30}$ kg

$$\lambda = \frac{6.6\times10^{-34}}{\sqrt{2\times10^{-30}\times10^{-18}}} \simeq 0.5 \text{ nm}$$

neutron: $m \sim 2\times10^{-27}$ kg

$\lambda \sim 0.01$ nm

3 What happens when we look to see where the electron went?



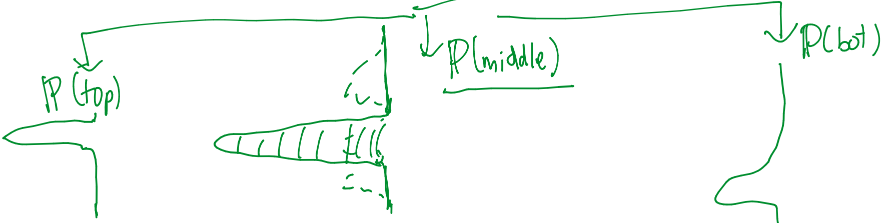
Where did it go?

- if e^- was point…
- quantum wave/interference:
  - electron not in one location:

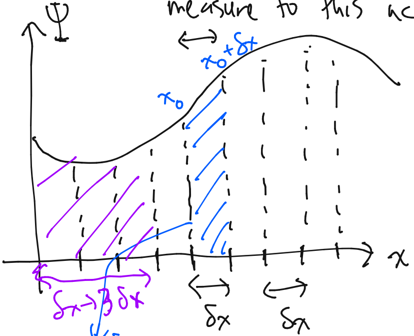
$$\mathbb{P}(\text{middle}) = \int_{\text{middle}} dV \, |\Psi|^2$$

(higher-d. → dV)

MEASURE location of e^-:



4 What happens to the wave function after a measurement?

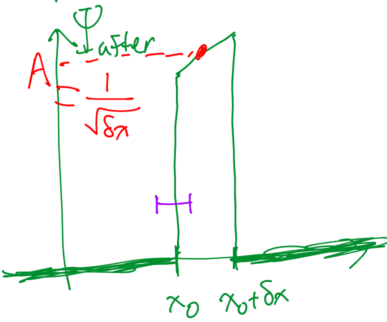


$\Psi \xrightarrow{\text{measurement}} \Psi_{after} = c \cdot \Psi \cdot \begin{cases} 1 & \text{meas. compatible} \\ 0 & \text{otherwise} \end{cases}$

(c: normalization const.)

$$P = \int_{x_0}^{x_0+\delta x} dx\, |\Psi|^2 \approx \delta x \cdot |\Psi(x_0)|^2$$

after I measure the particle to be here:



$$\delta x \cdot |A|^2 = \int_{x_0}^{x_0+\delta x} dx\, |A|^2 \simeq \int_{x_0}^{x_0+\delta x} dx\, |\Psi|^2_{after} = 1$$

(outside, $\Psi = 0$)

5 In popular science people often think that quantum mechanics is intrinsically random and unpredictable. Explain why this is (mostly) wrong.