PHYS 2170

General Physics 3 for Majors

Fall 2021

Lecture 29

Heisenberg's uncertainty principle

November 1

1 Review the quantum wave function. Decompose a quantum wave function into plane waves.

$\Psi(x)$: quantum wave function

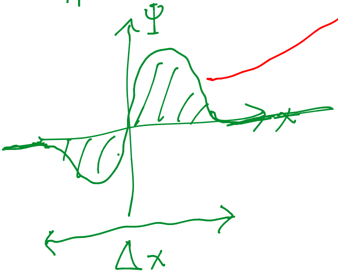
$p = \frac{h}{\lambda} = \hbar k$

$\mathbb{P}(a \le x \le b) \quad \int_a^b dx\, |\Psi|^2$

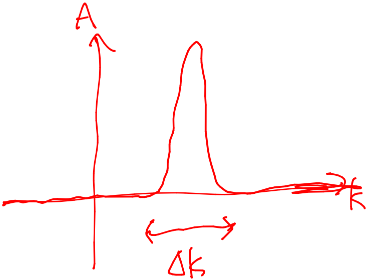
prob. upon measurement that...

$$\Psi(x) = \int_{-\infty}^{\infty} dk\, \underline{e^{ikx}}\, \underline{A(k)}$$

typical Ψ:



(uncertainty in x)



2 State and justify the position-momentum Heisenberg uncertainty principle.

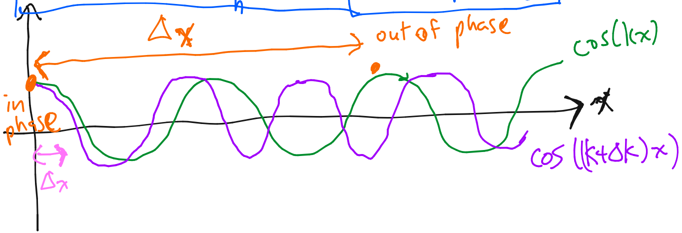
$$\boxed{\Delta x \, \Delta k \geq \frac{1}{2}}$$

Mathematics, not physics

physics → $k = \frac{p}{\hbar}$ $\therefore$ $\boxed{\Delta x \, \Delta p \geq \frac{\hbar}{2}}$

① k

② $k + \Delta k$



after distance Δx

$$\phi_1 = k \cdot \Delta x$$

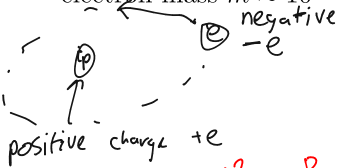
$$\phi_2 = (k + \Delta k)\Delta x$$

$$\phi_2 - \phi_1 = k\Delta x + \Delta k \cdot \Delta x - k\Delta x = \Delta x \cdot \Delta k = \pi$$

we see π phase diff whenever

$$\pi \leq \Delta x \cdot \Delta k$$

3 Estimate the binding energy of hydrogen, using $\hbar \approx 10^{-34}$ J · s, the electron mass $m \approx 10^{-30}$ kg, and $e^2/4\pi\epsilon_0 \approx 2 \times 10^{-28}$ J · m.



B

Kinetic energy: $p = mv$

$$K = \frac{1}{2} m v^2 = \frac{p^2}{2m}$$

potential energy:

prot elec

$$U = \frac{1}{r} \cdot \frac{1}{4\pi\epsilon_0} \cdot e \cdot (-e)$$

$$= -\frac{B}{r} \qquad B = \frac{e^2}{4\pi\epsilon_0}$$

total: $E = K + U = \dfrac{p^2}{2m} - \dfrac{B}{r}$

Heisenberg: $\Delta x \, \Delta p \geq \dfrac{\hbar}{2}$

$$E = \frac{\Delta p^2}{2m} - \frac{B}{\Delta x}$$

$$E \sim \frac{\Delta p^2}{2m} - \frac{B}{\hbar / 2\Delta p}$$

$$= \frac{\Delta p^2}{2m} - \frac{2B}{\hbar} \Delta p$$

$$\frac{\partial E}{\partial (\Delta p)} = 0$$

$$\frac{\Delta p}{m} - \frac{2B}{\hbar} = 0$$

$$\Delta p = \frac{2mB}{\hbar}$$

Minimize $E(\Delta p)$:

$$E = -2m \frac{B^2}{\hbar^2} \checkmark$$

$$= \left(\frac{2mB}{\hbar}\right)^2 \cdot \frac{1}{2m} - \frac{2B}{\hbar} \frac{2Bm}{\hbar}$$

$$\sim -8 \times 10^{-18} \text{ J}$$

$$\sim -50 \text{ eV}$$

4 The lifetime of the Higgs boson is $\sim 10^{-22}$ s. What is the uncertainty in its energy? What is the uncertainty in its mass (using relativity)?

time uncertainty: $\boxed{\Delta\omega \cdot \Delta t \geq 1/2}$ $\left(e^{ikx-i\omega t}\right)$

quantum: $\omega = \frac{1}{\hbar} E$

$\hookrightarrow \Delta E \, \Delta t \geq \frac{\hbar}{2}$

$\Delta t \sim 10^{-22}$ s

$\Delta E \sim \frac{\hbar}{2\Delta t}$

$\sim 5 \times 10^{-13}$ J

~ 3 MeV

$\Delta m = \Delta E \cdot \frac{1}{c^2}$

$m \sim 125 \; \frac{\text{GeV}}{c^2}$

$\Delta m \sim 3 \text{ MeV} \cdot \frac{1}{c^2}$