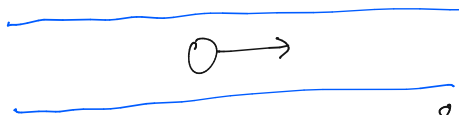**PHYS 2170**

General Physics 3 for Majors

Fall 2021

Lecture 3

Length contraction

August 27

1 The pion is a particle which decays in a time of order $\tau \approx 10^{-8}$ s. At a particle accelerator, a pion which is created during a collision might travel at speed $v = \beta c$, with $\beta = 1 - \boxed{10^{-6}}$. How long does an observer in the lab frame see the pion live?

Small number!
Taylor expand

pion decays at time $\tau \approx 10^{-8}$ s in rest frame of pion (itself)

we see the pion for a time... 7 µs, $\approx 7 \times 10^{-6}$ s.
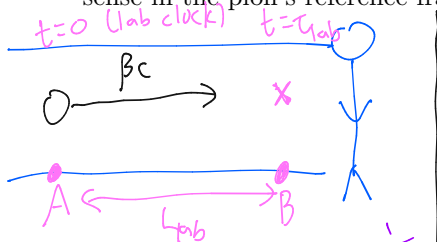
$$T_{lab} = \frac{T_{pion}}{\sqrt{1-\beta^2}}$$

$\beta = 1 - \delta \qquad (\delta \approx 10^{-6})$

$$\sqrt{1-\beta^2} = \sqrt{1-(1-\delta)^2}$$
$$= \sqrt{1-(1-2\delta+\delta^2)}$$
$$\approx \sqrt{2\delta}$$

$$T_{lab} = \frac{T_{pion}}{\sqrt{2\delta}} \rightarrow \frac{10^{-8} s}{\sqrt{2 \cdot 10^{-3}}} \approx 0.7 \times 10^{-5} s.$$

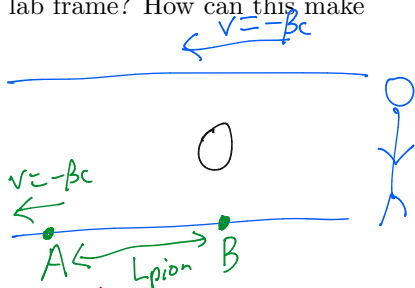**2** How far does the pion travel, in the lab frame? How can this make sense in the pion's reference frame?

$t=0$ (lab clock) $t=\tau_{lab}$

βc

X

A $\overset{\longleftarrow}{}$ B

L_{lab}

- pion decays $\gamma = \frac{1}{\sqrt{1-\beta^2}}$

  $\tau_{lab} \approx 7 \; \mu s$

  $\sim \gamma \tau_{pion}$

- Therefore pion travels distance

  $L_{lab} = (\beta c) \times \tau_{lab}$

  $= \beta \gamma c \, \tau_{pion}$

$v = -\beta c$

$v = -\beta c$

A $\overset{\longleftarrow}{}$ B

L_{pion}

- pion decays

  $\tau_{pion} \approx 10^{-8} \; s$

- so pion sees dist. as

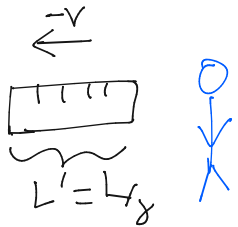 $L_{pion} = |velocity| \times time$

  $= \beta c \, \tau_{pion}$

  $L_{pion} = L_{lab} / \gamma$

3 Explain length contraction.

R

as seen as Blue:

$-v$

$L' = L/\gamma$

$[$ compare... $t' = \gamma(t)]$

time dilation implies
length contraction... vice versa.

L'

$[$ speed$_{R, \text{ seen } B}$ = speed$_{B, \text{ seen } R}]$

$$\frac{L_{R, \text{ seen } B}}{\Delta t_{R, \text{ seen } B}} = \frac{L_{B, \text{ seen } R}}{\Delta t_{B, \text{ seen } R}}$$

L

$t = \Delta t_{B, \text{ seen } R}$

$t' = \gamma t$

t

$t = 0$

B is keeping time

4 Is there transverse length contraction?

If then

Suppose...

from orange point of view...?

Contradictory! Thus, no transverse length contraction.

[I can change $x \rightarrow -x$]
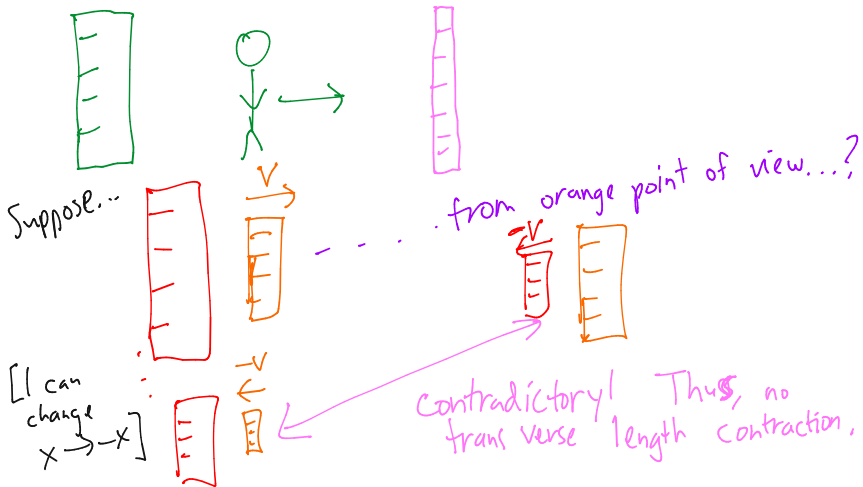
5 In its rest frame, a ruler is rotated by angle θ relative to the x-axis. Describe the ruler as viewed by an observer moving at velocity $v\hat{\mathbf{x}}$ (relative to the ruler frame).