PHYS 2170

General Physics 3 for Majors

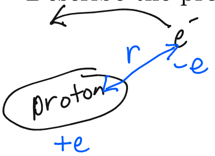
Fall 2021

Lecture 34

Bohr's model of the hydrogen atom

November 15

1 Describe the problem of quantizing the hydrogen atom.



Classically:

$$E = \frac{\vec{p}^{\,2}}{2m} + \frac{1}{4\pi\varepsilon_0}\frac{(+e)(-e)}{r}$$

electron mass

$$B = \frac{e^2}{4\pi\varepsilon_0}$$

$$\boxed{E = \frac{\vec{p}^{\,2}}{2m} - \frac{B}{r}}$$

"central force"

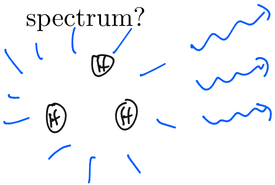
Write Schrödinger's Eqn:

allowed energy E:

$$E\Psi = \frac{\hbar^2}{2m}\left[-\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}\right]\Psi - \frac{B}{r}\Psi$$

- Work in spherical coords $\quad n = 1, 2, 3, \ldots$
- Separation of variables
- Allowed energies: $E_n = -\frac{mB^2}{2\hbar^2}\cdot\frac{1}{n^2}$ $\rightarrow$ 13.6 eV $\sim 2\times10^{-18}$ J

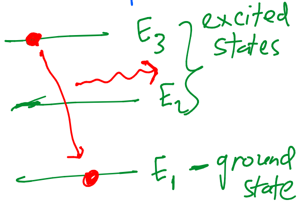
$\boxed{2}$ What was the historical observation about the hydrogen atom spectrum?



most light emitted at specific λ:

$$\underset{\text{emitted}}{\frac{1}{\lambda}} = \frac{1}{\underset{\text{const.}}{\lambda_*}}\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$$

$n_1 < n_2$



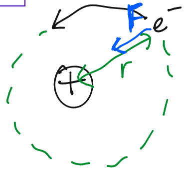
$$E_{ph} = E_3 - E_1$$
$$= -13.6\text{ eV}\left[\frac{1}{3^2}-\frac{1}{1^2}\right]$$
$$= 13.6\text{ eV}\left[1-\frac{1}{3^2}\right]$$

$$E_{ph} = h f_{ph} = \frac{hc}{\lambda_{ph}}$$

$$\frac{hc}{\lambda_{ph}} = (13.6\text{ eV})\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$$

$$\frac{1}{\lambda_*} = \frac{13.6\text{ eV}}{hc} \sim 100\text{ nm}$$

3 Describe the classical orbits of the electron in hydrogen.



$$F = m a_c$$

$$\frac{B}{r^2} = m\frac{v^2}{r}$$

$$B = mv^2 r$$

$$v^2 = \frac{B}{mr}$$

$$v = \sqrt{\frac{B}{mr}}$$

electron energy:

$$E = \frac{1}{2}mv^2 \boxed{-\frac{B}{r}}$$

potential energy U

$$\cancel{\vec{F} = -\nabla U}$$

$$\cancel{F_x = -\frac{\partial U}{\partial x}}$$

here: $F_r = -\frac{\partial U}{\partial r}$

$$F_r = -\frac{\partial}{\partial r}\left(\frac{-B}{r}\right)$$

$$= -\left(-B \cdot -\frac{1}{r^2}\right)$$

$$F = \frac{B}{r^2}$$

Exercise: r in terms of E.

Answer: $E = -\frac{B}{2r}$

$$\left[E = \frac{1}{2}m\left(\frac{B}{mr}\right) - \frac{B}{r}\right]$$

$$r = -\frac{B}{2E}$$

4 Apply Bohr's quantization condition.

Postulate: angular momentum $L_z = n\hbar$ is quantized. $n = 1, 2, 3, \ldots$

In classical model:

$$L_z = (mV)r$$

$$= m\sqrt{\frac{B}{mr}}\, r$$

$$= \sqrt{Bmr}$$

$$= n\hbar$$

Deduce: only orbits at

$$r_n = \frac{\hbar^2}{mB} n^2$$

$$= a_B \cdot n^2$$

Bohr radius $\sim 5 \times 10^{-11}$ m

Therefore, energy levels are:

$$E_n = -\frac{B}{2r_n} = -\frac{mB^2}{2\hbar^2} \cdot \frac{1}{n^2}$$

atom physics scales:

- electron mass $[10^{-30}\,\mathrm{kg}]$
- carbon atom mass $[2 \times 10^{-26}\,\mathrm{kg}]$
- binding energy $[\sim 10\,\mathrm{eV}]$
- Bohr radius $a_B \sim 0.05$ nm

$$\omega_{vib} \sim \sqrt{\frac{m_{el}}{m_{at}}}\, \omega_{el}$$

$$\omega_{rot} \sim \frac{m_{el}}{m_{at}}\, \omega_{el}$$

Chemistry:

binding/covalent bond: ~ 5 eV

length of bond: ~ 0.1 nm