PHYS 2170

General Physics 3 for Majors

Fall 2021

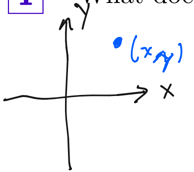
Lecture 38

Bosons and fermions

December 1

1 What does the wave function look like for two particles?

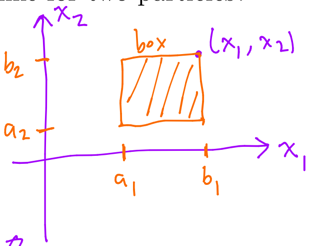
particle in 2d



[Classical info]

Quantum: $\Psi(x,y)$

- function of 2 coords

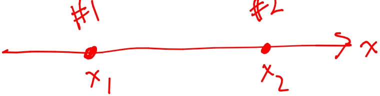


Quantum: $\Psi(x_1, x_2)$

$$\int_{box} dx_1 dx_2 \, |\Psi|^2 = \text{probability of measuring } a_1 \le x_1 \le b_1,\ a_2 \le x_2 \le b_2 .$$

equivalent!

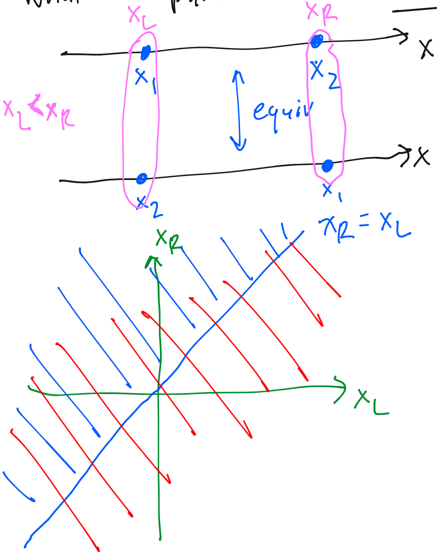
2 particles in 1d



Classical info: (x_1, x_2)

2 What is a boson? What is a fermion?

What if particles are indistinguishable?



$\Psi_A = \Psi(x_1, x_2)$

$\Psi_B = \Psi(x_2, x_1)$

Convenient to extend Ψ to whole plane, need Ψ_A is equiv to Ψ_B.

$\Psi_A = k\,\Psi_B$ ← constant

$\Psi(x_1,x_2) = k\,\Psi(x_2,x_1)$

$= k \cdot k\,\Psi(x_1,x_2)$

$= k^2\,\Psi(x_1,x_2)$

so $k^2 = 1$.

$k = 1$ → BOSON

$k = -1$ → FERMION.

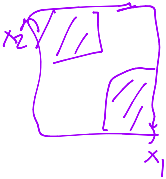
3 State and derive the Pauli exclusion principle.

Suppose 2 fermions: $\Psi(x_1, x_2) = -\Psi(x_2, x_1)$

[antisymmetric]

"exchange force"

$|\Psi|^2$



Suppose $x_1 = x_2 = x$

$\Psi(\overset{(x_1)}{x}, \overset{(x_2)}{x}) = -\Psi(\overset{(x_2)}{x}, \overset{(x_1)}{x})$

Therefore $\Psi(x, x) = 0$.

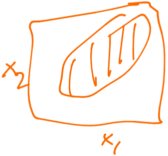
Pauli exclusion: 2 fermions Never be in same state [here $x_1 = x_2$].

Boson: $\Psi(x_1, x_2) = \Psi(x_2, x_1)$

$x_1 = x_2$?

$\Psi(x, x) = \Psi(x, x)$ does not, in general, $= 0$.

$|\Psi|^2$



4 Define the spin of a particle, and relate it to whether the particle is a boson or a fermion.

All particles have an intrinsic spin (angular momentum).

$\vec{S} = (S_x, S_y, S_z)$

[analogue: $\vec{L} = (L_x, L_y, L_z)$] $|m| \le \ell \sim s$

(e)

(s = ~~total~~ particle's spin) $\vec{S}^2 = \hbar^2 s(s+1)$

$S_z = \hbar m$

$m = -s, -s+1, \dots, +s$

When is well-defined?

Need integer k; $-s + k = +s$

$k = 2s$

$s = \frac{1}{2}k$

electron = spin-1/2.
(fermion)

internal: $m = 1/2$, $m = -1/2$

Allowed values of s: $0, \frac{1}{2}, 1, \frac{3}{2}, \dots$

BOSONS: $s = 0, 1, \dots$

FERMION: $s = 1/2, 3/2, \dots$

5 Describe the wave function of a particle with spin.

One Electron wave function: $\Psi(x, y, z; m)$ ← either $+1/2$ or $-1/2$.

(x, y, z: continuous coords.)

or: $\begin{pmatrix} \Psi_{+1/2}(x,y,z) \uparrow \\ \Psi_{-1/2}(x,y,z) \downarrow \end{pmatrix}$

2 electron wave functions: $\Psi(\underbrace{x_1, y_1, z_1, m_1}_{\text{particle 1}} / \underbrace{x_2, y_2, z_2, m_2}_{\text{particle 2}})$

shorthand → $\Psi(1, 2)$

Pauli exc: $\Psi(1,2) = -\Psi(2,1)$

$$\Psi(1,2) = \psi(x_1, y_1, z_1)\psi(x_2, y_2, z_2) \cdot \frac{1}{\sqrt{2}}(\uparrow_1 \downarrow_2 - \downarrow_1 \uparrow_2)$$