PHYS 2170

General Physics 3 for Majors

Fall 2021

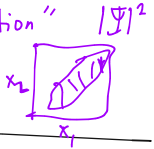
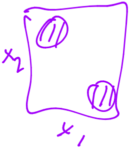
Lecture 39

The independent particle approximation

December 3

1 Review bosons and fermions.

2 types of indistinguishable particles in QM:

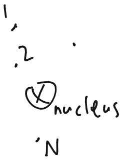
boson	spin $s=0,1,2,3,\dots$ $\vec{S}^2=\hbar^2 s(s+1)$	no restrictions on # of bosons in a state symmetric: $\Psi(1,2)=\Psi(2,1)$	$\Psi(x_1,x_2)=\psi(x_1)\phi(x_2)+\psi(x_2)\phi(x_1)$ "apparent interaction" 
fermions	spin $s=\frac{1}{2},\frac{3}{2},\frac{5}{2},\dots$	Pauli exclusion: no 2 fermions occupy same state antisymmetric $\Psi(1,2)=-\Psi(2,1)$	$\Psi(x_1,x_2)=\psi(x_1)\phi(x_2)-\psi(x_2)\phi(x_1)$ "repulsion" 

$\boxed{2}$ What is the independent particle approximation?

"unwanted"

$$E = K_1 + \cdots + K_N - \frac{(Ne)e}{4\pi\varepsilon_0 r_1} - \cdots - \frac{Ne^2}{4\pi\varepsilon_0 r_N} + \boxed{\frac{e^2}{4\pi\varepsilon_0 |r_1 - r_2|} + \cdots + \frac{e^2}{4\pi\varepsilon_0 |r_{N-1} - r_N|}}$$

$$K_1 \rightarrow \frac{p_1^2}{2M} \quad \text{(kinetic)}$$



Z (or N) electrons

- begin by ignore interactions btwn electrons

$$E = \underbrace{\left[\frac{p_1^2}{2m} - \frac{Ze^2}{4\pi\varepsilon_0 r_1}\right]}_{E_1} + \underbrace{\left[\frac{p_2^2}{2m} - \frac{Ze^2}{4\pi\varepsilon_0 r_2}\right]}_{E_2} + \cdots$$

- if we know allowed E's of 1 particle,

$$E_{total} = E_{\#1} + E_{\#2} + \cdots$$

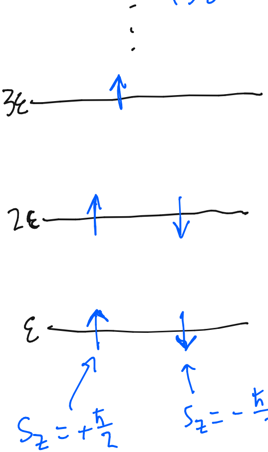
caveat: if fermions, Pauli exclusion must be obeyed.

3 A quantum system has single-particle energy levels $\epsilon, 2\epsilon, 3\epsilon, \ldots$. What is the ground state energy of 5 particles of spin $\frac{1}{2}$, 1, or $\frac{3}{2}$?

spin - $1/2$

fermions:

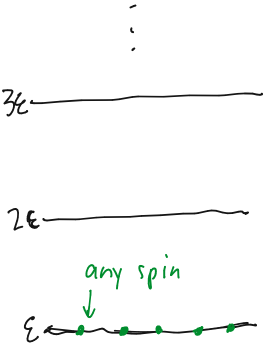
total: $\varepsilon+\varepsilon+2\varepsilon+2\varepsilon+3\varepsilon = 9\varepsilon$



Spin 1

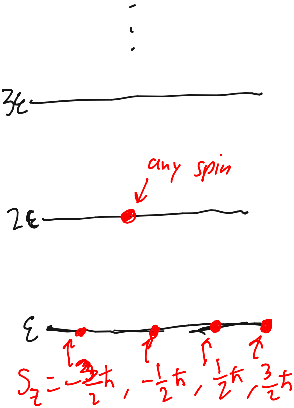
total energy = 5ε

bosons

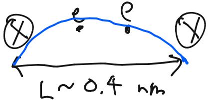


Spin $\frac{3}{2}$.

total energy = 6ε



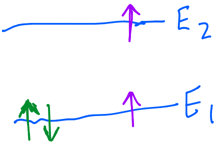
4 Consider 2 electrons of mass $m \approx 10^{-30}$ kg in a solid between two atoms separated by $L \approx 4 \times 10^{-10}$ m. Suppose the electrons are modeled well by a "particle in a box" of length L. What is the ground state energy if the electrons have opposite/same spin?



quantum "particle in a box" of length L:

$k_n = \frac{n\pi}{L}$ allowed wavelengths: $\lambda_n = 2L/n$ $n = 1, 2, 3, \ldots$

$(p = \hbar k)$ $E_n = \frac{p^2}{2m} = \frac{\hbar^2}{2m} k_n^2 = \frac{\hbar^2 \pi^2}{2mL^2} n^2$



Opposite S_z: $E_{min} = 2E_1 = 2\frac{\hbar^2\pi^2}{2mL^2}$

$\boxed{E_1 \sim 3\times10^{-19}\ \mathrm{J}}$

Same S_z: $E_{min} = E_1 + 4E_1$
$= 5\frac{\hbar^2\pi^2}{2mL^2}$

Do you think the electrons will be closer or farther in the state with the same spin, or in the state with opposite spin?

Spatial wave function is antisym if both ↑: apparent repulsion $|\Delta x_{12}| \sim L/2$

opp spin: Sym spatial wave function; $|\Delta x_{12}| \sim L/4$

5 Using that

$$\frac{e^2}{4\pi\epsilon_0} \sim 2 \times 10^{-28}\ \mathrm{J \cdot m}, \qquad L \sim 4 \times 10^{-10}\ \mathrm{m}$$

estimate the *loss* in potential energy ΔU if the electrons are in the same-spin state, and the *gain* in kinetic energy. Which one wins out?

$$\Delta U = U_{same} - U_{opp} = \frac{e^2}{4\pi\varepsilon_0 |\Delta x_{same}|} - \frac{e^2}{4\pi\varepsilon_0 |\Delta x_{opp}|}$$

$$= \frac{e^2}{4\pi\varepsilon_0}\left[\frac{1}{L/2} - \frac{1}{L/4}\right] = -\frac{2}{L}\frac{e^2}{4\pi\varepsilon_0} \sim -1 \times 10^{-18}\ \mathrm{J}$$

$$\Delta K = K_{same} - K_{opp} \sim 0.9 \times 10^{-18}\ \mathrm{J} \sim 3 \cdot \underbrace{\frac{\hbar^2\pi^2}{2mL^2}}_{0.3\times 10^{-18}}$$

$$(\Delta U + \Delta K) = -10^{-19}\ \mathrm{J}$$

$(\sim 5000\mathrm{K})$

: Spontaneous aligned spin

$\downarrow$

magnetism in iron...