PHYS 2170

General Physics 3 for Majors

Fall 2021

Lecture 4

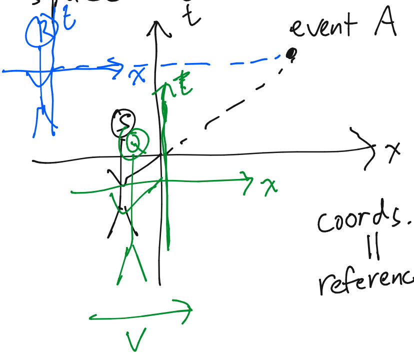
Lorentz transformations

August 30

1 Explain the Lorentz transformations.

spacetime coords: event (t, x, y, z)

coords. are relative



coords.
||
reference frame

If S observes event A at $(t, x)_S$, what does Q find: $(t, x)_Q$?

Observation #1: t & x have units.

$(t,x)_S \longrightarrow (t,x)_Q$ obey [linearity]

$(\lambda t, \lambda x)_S \longrightarrow (\lambda t, \lambda x)_Q$

Observation #2: choice of origin not important.

2 Argue that $-(c\Delta t)^2 + \Delta x^2$ is independent of frame. Use this fact to argue for the form of the Lorentz transformations.

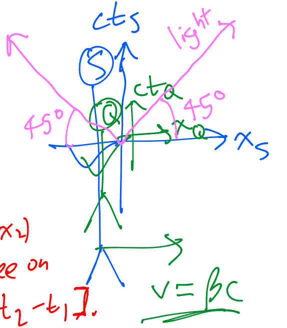
Lorentz transformations:

$$ct_Q = a_1 ct_S + a_2 x_S$$

$$x_Q = a_3 ct_S + a_4 x_S$$

$a_1, \ldots, a_4$ constants [dimensionless]

linear transformation,



Postulate #2 (revised): if (ct_1, x_1) & (ct_2, x_2) denote 2 events... inertial frames agree on

$\underbrace{(\Delta x)^2 - (c\Delta t)^2}_{=0 \text{ for light}}$ $[\Delta x = x_2 - x_1,\ \Delta t = t_2 - t_1]$.

drop Δ: $x_S^2 - c^2 t_S^2 = x_Q^2 - c^2 t_Q^2 = (a_3 ct_S + a_4 x_S)^2 - (a_1 ct_S + a_2 x_S)^2$

complete square... organize terms by x_S^2, t_S^2, ...

$$0 = a_3 a_4 - a_1 a_2$$

$$1 = a_4^2 - a_2^2 = a_1^2 - a_3^2$$

family of sol'ns: $a_1 = a_4 = \frac{1}{\sqrt{1-\beta^2}}$; $a_2 = a_3 = \frac{\beta}{\sqrt{1-\beta^2}}$ $\quad |\beta| < 1$

$\boxed{3}$ Consider events $(ct_A, x_A) = (1,3)$, $(ct_B, x_B) = (5,5)$, $(ct_C, x_C) = (2,8)$. Find a pair of events that are timelike, spacelike, and lightlike separated.

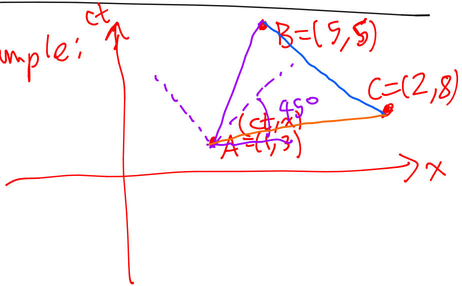
Summary: if frame S' moves at velocity v relative to S:

(t', x') $\qquad\qquad$ (t, x)

$$ct' = \gamma(ct - \beta x)$$

$$x' = \gamma(x - \beta ct)$$

Example:



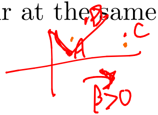
- LIGHTLIKE separated if $\Delta x^2 - c^2\Delta t^2 = 0$
- TIMELIKE sep. if $\Delta x^2 - c^2 \Delta t^2 < 0$ $= -c^2\Delta\tau^2 \leftarrow$ "proper time"
- SPACELIKE if $\Delta x^2 - c^2\Delta t^2 > 0$.

$$\Delta x = x_B - x_C = 5 - 8 = -3$$

$$c\Delta t = ct_B - ct_C = 5 - 2 = 3$$

$$\underbrace{(-3)^2}_{\Delta x^2} - \underbrace{3^2}_{c^2\Delta t^2} = 0$$

$\boxed{4}$ Consider events $(ct_A, x_A) = (1,3)$, $(ct_B, x_B) = (5,5)$, $(ct_C, x_C) = (2,8)$. For the pair that is spacelike separated, determine the velocity of the reference frame at which they occur at the same time.



A&C spacelike separated:
no cause and effect

$$c\Delta t' = \gamma(c\Delta t - \beta\Delta x) \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\Delta x' = \gamma(\Delta x - \beta c\Delta t)$$

A & C simultaneously:

$$\Delta t' = 0$$

$$c\Delta t = \beta\Delta x$$

$$c\Delta t = 2-1 = c(t_C - t_A)$$

$$\Delta x = 8-3 = 5$$

$$\beta = \frac{c\Delta t}{\Delta x} = \frac{1}{5} < 1.$$

$$v = c/5$$

If $\beta = \frac{v}{c} > 1/5$, then $\Delta t' < 0$
[C before A]

while $\beta < 1/5$, then $\Delta t' > 0$
[A before C]

5 Use the Lorentz transformations to explain time dilation and length contraction.