PHYS 2170

General Physics 3 for Majors

Fall 2021

Lecture 40

The periodic table

December 6

1 Review bosons and fermions, and the independent particle approximation.

2 types of <u>indistinguishable</u> particles

			INDEPENDENT:
bosons	spin $S=0, 1, 2, 3, \ldots$	no exclusion	ground state: put all bosons in lowest energy single-p. state
fermions	spin $S=\frac{1}{2}, \frac{3}{2}, \ldots$	Pauli excl: 2 fermions can't be in same state [both position/orbital and spin]	add fermions one at a time. - start w/ lowest E state unoccupied - make sure all states used.

2 Review the hydrogen atom spectrum, and describe the multi-electron atom problem.

energy levels in H atom:

$$E_{n\ell m} = -\frac{13.6\text{ eV}}{n^2}$$

$$\vec{L}^2 = L_x^2 + L_y^2 + L_z^2 = \hbar^2 \ell(\ell+1)$$

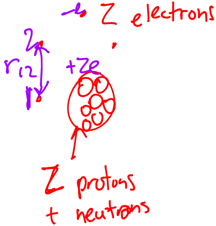
$\ell = 0, 1, 2, \ldots < n$

$L_z = \hbar m: \quad |m| \le \ell$

$m = -\ell, \ldots, +\ell$

if nucleus has $+Ze$:

$$E_{n\ell m} = \frac{-13.6}{n^2} \times Z^2$$



$$E = \frac{p_1^2}{2M} + \frac{p_2^2}{2M} + \cdots + \frac{p_Z^2}{2M} - \frac{Ze^2}{4\pi\varepsilon_0 r_1} - \frac{Ze^2}{4\pi\varepsilon_0 r_2} \cdots - \frac{Ze^2}{4\pi\varepsilon_0 r_Z}$$

$$+ \frac{e^2}{4\pi\varepsilon_0 r_{12}} + \cdots$$

elec-elec repulsion

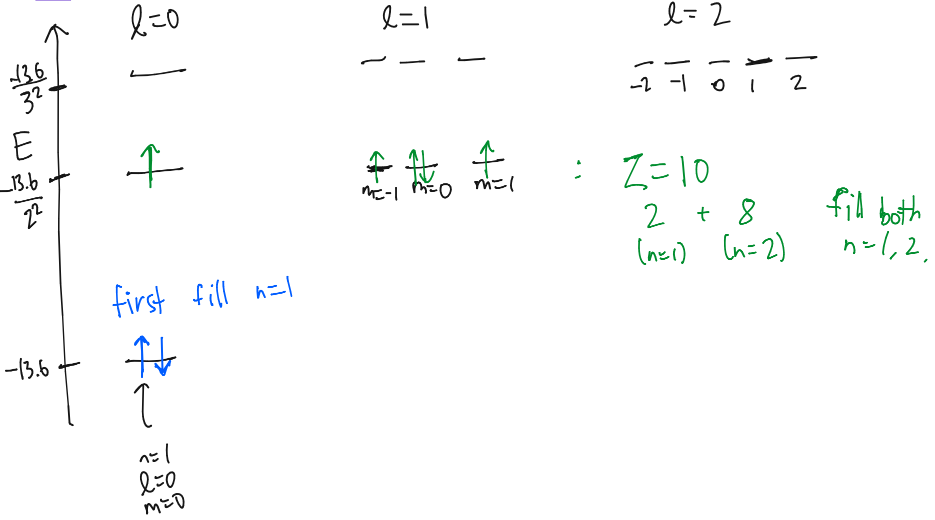
try to ignore them...

single-particle

$$E = \left[\frac{p_1^2}{2M} - \frac{Ze^2}{4\pi\varepsilon_0 r_1}\right] + \cdots + \left[\frac{p_Z^2}{2M} - \frac{Ze^2}{4\pi\varepsilon_0 r_Z}\right]$$

$$E = E_1 + E_2 + \cdots + E_Z$$

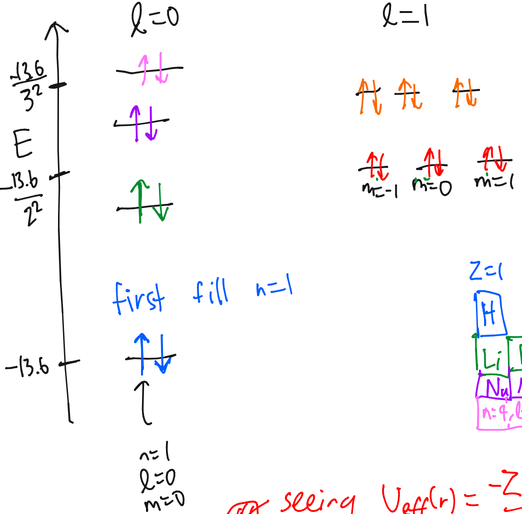
3 What is the periodic table you would predict?

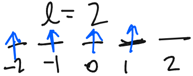


↑ : $S_z = 1/2$

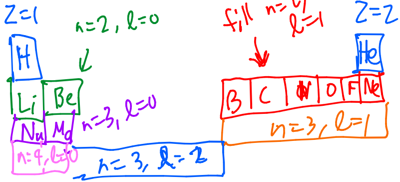
↓ : $S_z = -1/2$

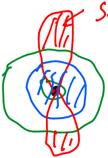
4 What is the actual structure of the periodic table. Why does it differ?





$$E_{3,2,m} > E_{4,0,0}$$





seeing $V_{eff}(r) = \frac{-Z_{eff}(r)\, e^2}{4\pi\varepsilon_0 r}$

5 Estimate the size of an atom with Z electrons.

Recall: in our problem, modifying Bohr model;

$$E_n \sim -\frac{Ze^2}{4\pi\epsilon_0 r_n}$$

$$E_n = -Z^2 \cdot \frac{13.6}{n^2} \qquad n = 1, 2, 3, \ldots$$

$$r_n \sim n^2 r_1 \qquad r_n \sim \frac{n^2}{Z} r_1$$

$$r_n \sim \frac{n_{max}^2}{1} a_B$$

$$r_n \sim Z^{2/3}$$

- Every time we add an electron:
  - fill up one (n, ℓ, m, s_z)
  - $Z \to Z-1$

Largest value of n?

Lec 37: at fixed n, $2n^2$ available states.

Want $Z \sim 2 + 2\cdot 2^2 + 2\cdot 3^2 + \ldots +$ change

$$Z \sim 2\int_0^{n_{max}} dk \cdot k^2 \sim \frac{2}{3} n_{max}^3$$

$$n_{max} \sim Z^{1/3}$$