PHYS 2170

General Physics 3 for Majors

Fall 2021

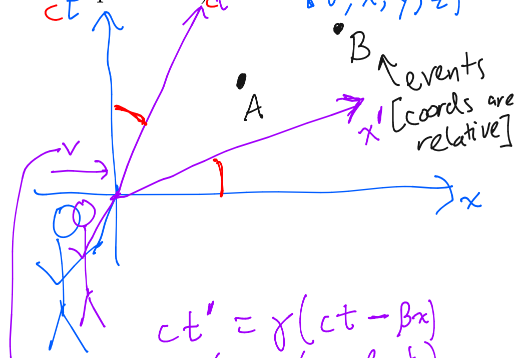
Lecture 5

Pole-in-barn paradox

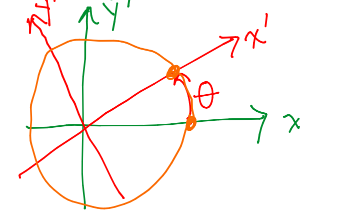
September 1

1 Review the Lorentz transformations (and extend them to 4D spacetime).

(t, x, y, z)



Analogy: rotations



$$ct' = \gamma(ct - \beta x)$$
$$x' = \gamma(x - \beta ct)$$
$$y' = y$$
$$z' = z$$

no tranverse length contract.

$$\beta = \frac{v}{c}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$x^2 - c^2t^2$ invariant

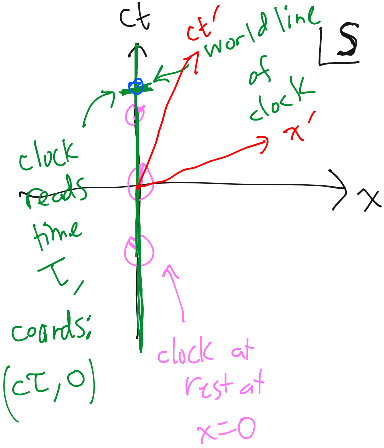
$$x' = \cos\theta\, x + \sin\theta\, y$$
$$y' = -\sin\theta\, x + \cos\theta\, y$$

$x^2 + y^2$ invariant

2 Use the Lorentz transformations to explain time dilation.

time dilation

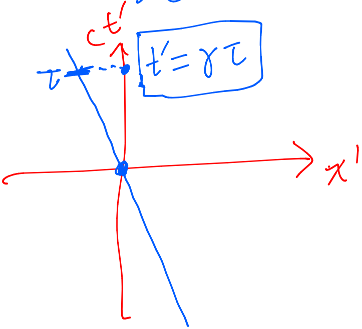
clock at rest in frame S



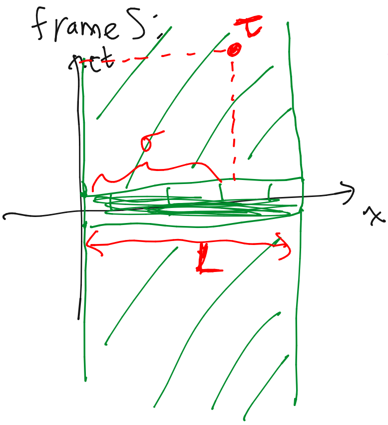
frame S′ moves at velocity $v = c\beta$ rel. to S:

$$ct' = \gamma(c\tau - 0)$$

$$x' = \gamma(0 - \beta c\tau)$$



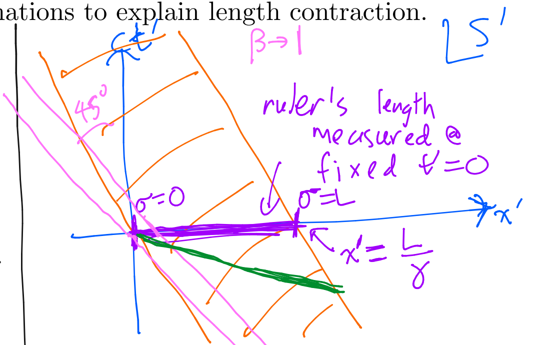
3 Use the Lorentz transformations to explain length contraction.



ruler exists when

$0 < \sigma < L$

$-\infty < \tau < \infty$



$$ct' = \gamma(c\tau - \beta\sigma) = 0$$

$$x' = \gamma(\sigma - \beta c\tau)$$

$$c\tau = \beta\sigma$$

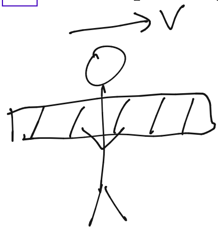
$$x' = \gamma(\sigma - \beta\cdot\beta\sigma)$$

$$= \gamma(1-\beta^2)$$

$$= \sigma/\gamma$$

$$\left[\gamma = \frac{1}{\sqrt{1-\beta^2}}\right]$$

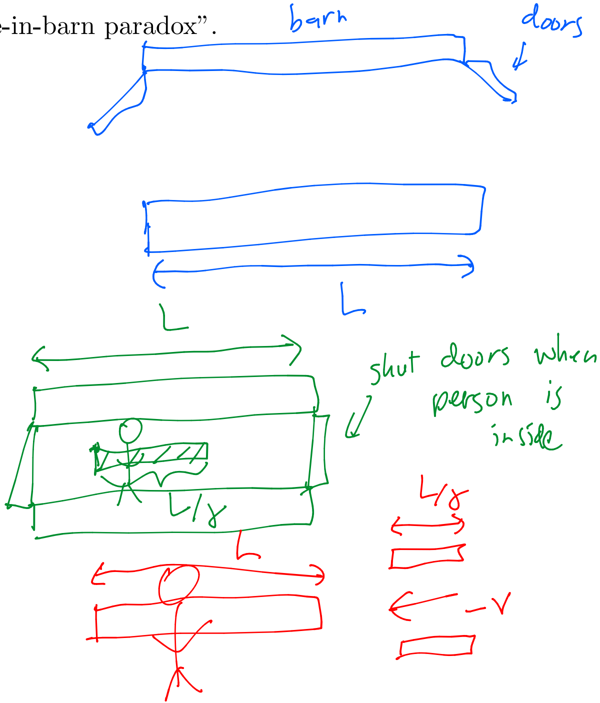
4 Set up the "pole-in-barn paradox".



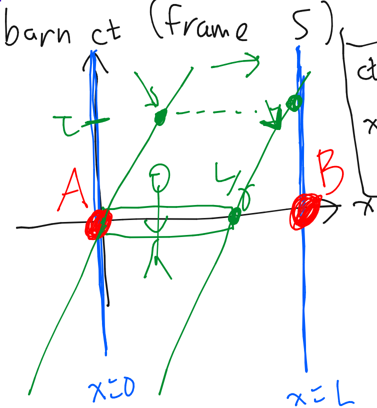
if pole is at rest, it has length L

barn sees:

runner sees:



5 Resolve the "pole-in-barn paradox".



$$ct' = \gamma(ct - \beta x)$$
$$x' = \gamma(x - \beta ct)$$

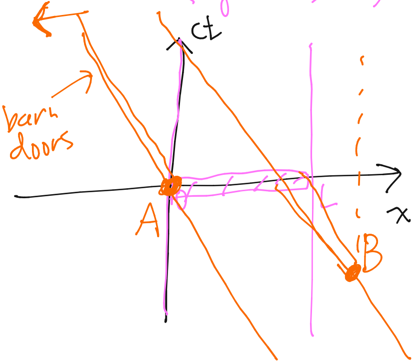
runner (frame S')

A: $(ct', x') = (0, 0)$

B: $(ct', x') = (-\gamma\beta L, \gamma L)$

left edge: $\left(\frac{c\tau}{\gamma}, 0\right)$

right edge: $\left(\frac{c\tau}{\gamma} - \beta L, L\right)$



A: $(ct, x)_S = (0, 0)$

B: $(ct, x)_S = (0, L)$

left: $(ct, x) = (c\tau, \beta c\tau)$

right: $(ct, x) = \left(c\tau, \beta c\tau + \frac{L}{\gamma}\right)$