# PHYS 2170

## General Physics 3 for Majors

## Fall 2021

# Lecture 6

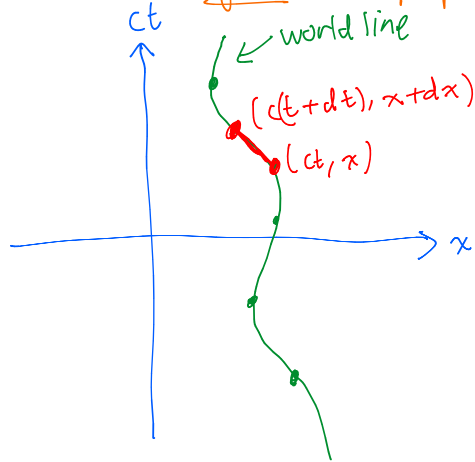
## Relativistic velocity addition

September 3

$\boxed{1}$ Define $\underbrace{\text{proper time}}$.

time a particle sees elapsed in its own rest frame
[time a clock reads]

ALL frames agree on proper time.




what proper time $d\tau$ has elapsed?

- All frames agree on
$$\Delta x^2 - c^2\Delta t^2$$
$$dx^2 - c^2 dt^2$$

- If particle were at rest:
$$dx = 0.$$
$$dt = d\tau$$

- $$\underline{-c^2 d\tau^2 = -c^2 dt^2 + dx^2}$$

**2** Define a natural notion of relativistic 4-velocity that transforms nicely under Lorentz transformations.

Suppose worldline is $(ct(\tau), x(\tau), y(\tau), z(\tau))$

Because $\tau$ is agreed on by all inertial frames, Lorentz transform is "the same": $S \to S'$ (moving vel. $\vec{v} = c\beta\hat{x}$)

NOT der.

$$\vec{c\,t'}(\tau) = \gamma\left(ct(\tau) - \beta x(\tau)\right)$$
$$x'(\tau) = \gamma\left(x(\tau) - \beta c t(\tau)\right)$$
$$y'(\tau) = y(\tau)$$
$$z'(\tau) = z(\tau)$$

Velocity $= \dfrac{\text{dist.}}{\text{time}} \times$ (vector direction)

Is there a better notion?

Because $\tau$ is invariant:

$$U_t = c\frac{dt}{d\tau}$$
$$U_x = \frac{dx}{d\tau}$$
$$U_y = \frac{dy}{d\tau}$$
$$U_z = \frac{dz}{d\tau}$$

$(U_t, U_x, U_y, U_z)$ is called 4-velocity.

$\dfrac{d}{d\tau}$

Lorentz transforms of 4-vel:

$$U_t' = c\frac{dt'}{d\tau} = \gamma(U_t - \beta U_x)$$
$$U_x' = \gamma(U_x - \beta U_t)$$
$$U_y' = U_y$$
$$U_z' = U_z$$

**3** Relate 4-velocity to the ordinary velocity vector $(u_x, u_y, u_z)$, and thus obtain the relativistic velocity addition rules.

The ordinary velocity vector:

$$u_x = \frac{dx}{dt} \qquad u_y = \frac{dy}{dt} \qquad u_z = \frac{dz}{dt}$$

If $U_x = \frac{dx}{d\tau}$ and $U_t = c\frac{dt}{d\tau}$

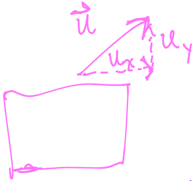
then $u_x = \frac{dx}{dt} = \frac{dx/d\tau}{dt/d\tau} = \frac{U_x}{(U_t/c)}$

$$u'_x = \frac{U'_x}{(U'_t/c)} = \frac{\gamma(U_x - \beta U_t)}{\frac{\gamma}{c}(U_t - \beta U_x)}$$

mult. $\frac{c/\gamma U_t}{c/\gamma U_t}$

$$= \frac{c\,U_x/U_t - \beta c}{1 - \beta\, U_x/U_t}$$

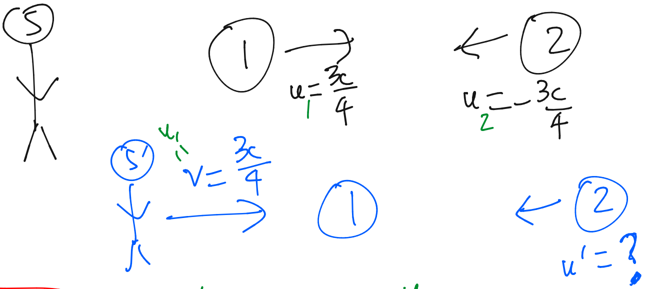
$$u'_x = \frac{u_x - v}{1 - \frac{v u_x}{c^2}}$$


$$u'_y = \frac{U'_y}{U'_t/c} = \frac{c U_y}{\gamma(U_t - \beta U_x)}$$

$$u'_y = \frac{c\,U_y/U_t}{\gamma\left(1 - \beta\frac{U_x}{U_t}\right)} = \frac{u_y}{\gamma\left(1 - \frac{v u_x}{c^2}\right)}$$

**4** In frame $S$, 2 balls move towards each other at velocity $v = 3c/4$. What velocity do the balls see each other traveling at?




Aside: in frame S, ball 2 moves at vel of $-\frac{3}{2}c$ RELATIVE to 1.

NOT bad.

$$u' = \frac{u_2 - u_1}{1 - \frac{u_1 u_2}{c^2}} = c\,\frac{-3/4 - 3/4}{1 - \frac{3}{4}\left(-\frac{3}{4}\right)}$$

$$= c\,\frac{-3/2}{1 + \frac{9}{16}} = c\,\frac{-3 \cdot \frac{1}{2} \cdot 16}{16 + 9} = -\frac{24}{25}c$$

**5** Show that it is not possible for velocities to exceed $c$ under relativistic velocity addition.