PHYS 2170

General Physics 3 for Majors

Fall 2021

Lecture 8

Massless particles

September 10

1 Review relativistic energy and momentum.

4-momentum $\left(p_t = \frac{E}{c}, p_x, p_y, p_z\right)$

4-velocity $U_t, U_x, \ldots$

$p_t = m U_t$

Nice Lorentz transformations! Frame S' moving at velocity $\vec{v} = c\beta\hat{x}$ [rel. to orig]:

$$p_t' = E'/c = \gamma(E/c - \beta p_x)$$

$$p_x' = \gamma(p_x - \beta E/c)$$

$$p_y' = p_y$$

$$p_z' = p_z$$

$\left[\gamma = \frac{1}{\sqrt{1-\beta^2}}\right]$

$(\beta \to -\beta)$

$$E^2 = (cp)^2 + (mc^2)^2$$

in all frames:

$$E^2 - (cp)^2 = (mc^2)^2$$

$$c^2\Delta t^2 - \Delta x^2 = c^2\Delta\tau^2$$

Start w/ particle at rest:

$E = mc^2$

$p_x = 0$

$p_y = 0$

$p_z = 0$

frame moving at velocity $-c\beta$ →

Particle moving at velocity $\vec{v} = c\beta\hat{x}$

$E = \gamma mc^2$ $\quad p_y = 0$

$p_x = \gamma mc\beta = \gamma m v$ $\quad p_z = 0$

2 What happens if $m = 0$?

$$E^2 = (cp)^2 + (mc^2)^2 \quad (m = 0)$$

$$E = \pm cp$$

A massless particle

$$\boxed{v = c\left(\frac{cp}{E}\right)} = \frac{\gamma m v \cdot c^2}{\gamma m c^2} \quad (\text{if } m \neq 0)$$

moves at speed of light $|v| = c$

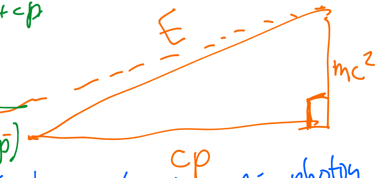
Can sign of momentum $p = \pm \frac{E}{c}$ change from one frame to another?

NO: always move left or right [along x-axis].

$E' = \gamma(E - \beta c p_x)$ etc. . . $\quad p = \gamma m v$

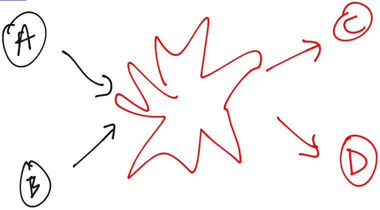
smooth funct. of v

$$\frac{E'}{cp'} = \frac{\gamma(E - \beta pc)}{\gamma(pc - \beta E)} = \frac{\gamma(cp - \beta cp)}{\gamma(cp - \beta cp)} = 1 \quad (E = +cp)$$



Physical example of $m=0$: photon (light)

3 Energy and momentum are conserved in relativity.



$$E_A + E_B = E_C + E_D$$

[conservation of energy]

$$\vec{p}_A + \vec{p}_B = \vec{p}_C + \vec{p}_D \checkmark$$

↑ x, y, z-momentum conservation

incoming, outgoing

$$\underbrace{E'_A + E'_B}_{\text{incoming}} - \underbrace{E'_C - E'_D}_{\text{outgoing}} = \gamma \underbrace{\left[E_A + E_B - E_C - E_D\right]}_{=0 \text{ [conservation]}}$$

$$- \gamma \beta c \underbrace{\left[p_{A,x} + p_{B,x} - p_{C,x} - p_{D,x}\right]}_{=0}$$

4 The electron and its antiparticle (the positron) have a mass of about $0.5\ \mathrm{MeV}/c^2$. If two photons collide together to create an electron-positron pair, how much energy must each photon have?

$+p > 0$ $\quad -p$

photons

$E = cp \quad [m = 0]$

electron $\quad$ positron [antiparticle]

e^- $\quad$ e^+

$(E', -p') \quad (E', p')$

momentum conservation

init: $p - p = 0 = p' - p'$

needed energy?

$E + E = E' + E'$

$E = E' = \gamma mc^2 \geq mc^2$

$m \approx 10^{-30}$ kg

$0.5 \dfrac{\mathrm{MeV}}{c^2}$

$E = 0.5 \dfrac{\mathrm{MeV}}{c^2} \times c^2$

Mass NOT conserved

energy !

5 Particle A (mass $m_{\mathrm{A}} > 0$) can transform into particle B (mass $m_{\mathrm{B}} > 0$) by emitting a single photon. Which one has higher mass?