PHYS 2170

General Physics 3 for Majors

Fall 2021

Lecture 9

Relativistic collisions

September 13

1 Review the conservation of energy and momentum.

- energy conservation
- momentum conserv.

$E_A + E_B = E_C + E_D$

incoming outgoing

E in these formulas = rest + kinetic

S'

(Lorentz transformations: S' moving @ rel. velocity $c\beta\hat{x}$ to S

$$\frac{E'}{c} = \gamma\left(\frac{E}{c} - \beta p_x\right)$$ ← frame S

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

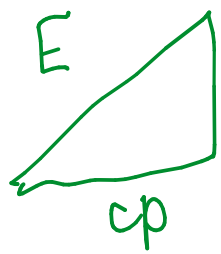
$$p_x' = \gamma\left(p_x - \beta\frac{E}{c}\right)$$

$$p_y' = p_y$$

$$p_z' = p_z$$

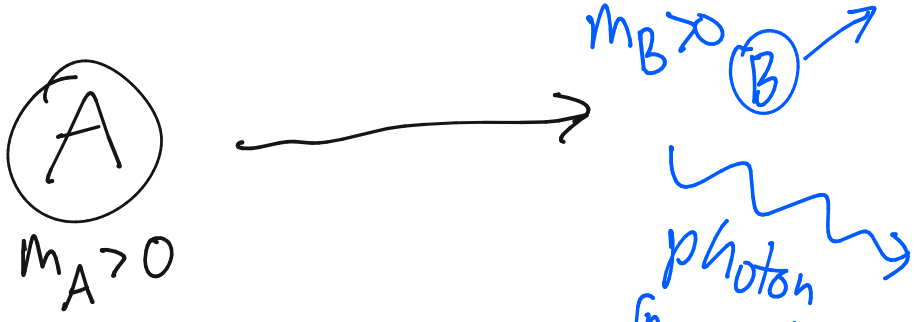
one particle of mass m:

$$E^2 - c^2(p_x^2 + p_y^2 + p_z^2) = (mc^2)^2$$



"Pythagorean relation"

2 Particle A (mass $m_A > 0$) can transform into particle B (mass $m_B > 0$) by emitting a single photon. Which one has higher mass?



$m_A > m_B$

$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$

Rest frame of A:

$E_{bef} = m_A c^2$

$p_{bef} = 0$

good frame: "center of mass" frame: total $p = 0$.

after break-up: $(E_{ph} > 0)$

$E_{aft} = \gamma_B m_B c^2 + E_{ph} = m_A c^2$

$p_{after} = 0$

$= \gamma_B m_B v_B - \frac{E_{ph}}{c}$

$p_{ph} = -\frac{E_{ph}}{c}$

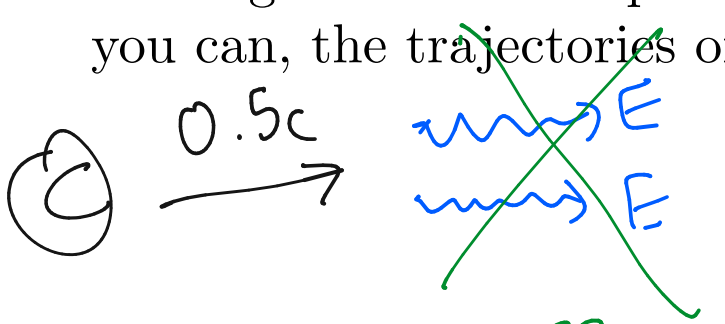
$m_A c^2 > \gamma_B m_B c^2$

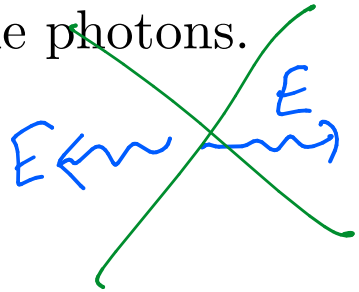
$m_A > \gamma_B m_B \quad \gamma_B > 1$

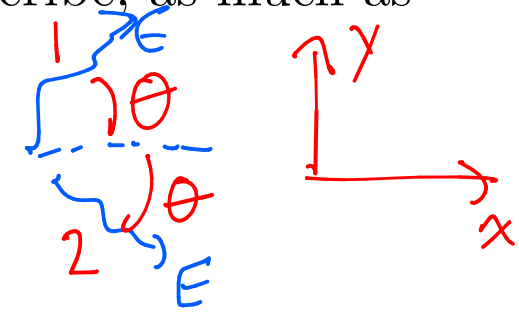
$m_A > m_B + (\gamma_B - 1) m_B$

$\boxed{m_A > m_B}$

3 Particle C is traveling at velocity $v = 0.5c$, when it suddenly disintegrates into two photons of equal energy. Describe, as much as you can, the trajectories of the photons.







$E_c = cp_c$ [only $m_c = 0$]

$$\begin{cases} p_{tot} = \dfrac{2E}{c} \\ E_{tot} = 2E \end{cases}$$

$$p_{tot} = \frac{E}{c} - \frac{E}{c} = 0$$

$$\begin{cases} p_{x1} = \dfrac{E}{c}\cos\theta \\ = p_{x2} \end{cases}$$

$$p_{y1} = \frac{E}{c}\sin\theta$$

$$p_{y2} = -\frac{E}{c}\sin\theta$$

energy conservation:

before: $\gamma m_c c^2$ = after: $2E$

x-mom conservation:

$$\gamma m_c v = \frac{2E\cos\theta}{c}$$

$v \uparrow c/2$

$$\frac{cp_{x,tot}}{E_{tot}}$$

$c/2$

$$\frac{\gamma m_c cv}{\gamma m_c c^2} = \frac{2E\cos\theta}{2E} = \cos\theta$$

$$1/2 = \cos\theta$$

$\theta = 60^\circ = \pi/3$

4 A very heavy particle of mass M, traveling at speed $0.6c$, collides into a very light particle of mass $m \ll M$, at rest. After the collision, estimate the speed of each particle.