PHYS 2170: General Physics 3 for Majors **Fall 2021**

Practice Exam 1

- This exam is open book and open notes. You are not allowed to use any electronic devices, nor allowed to communicate in any way about the exam with any other students until 24 hours after the exam is over.
- You have 2 hours to complete this exam (unless Disability Services has authorized a request for extended time). Good luck!

25 **Problem 1:** In inertial reference frame R, consider the following 3 events which occur at the following points in spacetime (measured in light years):

$$A: \quad (ct, x) = (0, 0), \qquad B: \quad (ct, x) = (4, 3), \qquad C: \quad (ct, x) = (3, -3). \tag{1}$$

1.1. If two of these events lie along the worldline of a photon (propagating through free space), which two are they? Why?

1.2. If two of these events lie along the worldline of a massive spaceship, which two are they? Why?

1.3. Is it possible for something at event B to influence event C? Why or why not?

20 **Problem 2 (Heavy nucleus in a collider):** At the heavy ion collider in Brookhaven, NY, a gold ion is accelerated such that it is eventually traveling at a constant velocity $c\beta$.

2.1. If the experimentalist sees the moving ion has 1% of the volume of a stationary ion, how fast is the ion moving?

2.2. If the gold atom has rest mass m, how much kinetic energy does the experimentalist observe this particular moving ion has? Express your answer in terms of m and c, and numerical factors.

25 **Problem 3 (Compton scattering):** Suppose an incident photon of energy E, traveling in the $+x$ direction, strikes a particle of mass m at rest. After the collision, a photon of energy E' is detected, moving at a relative angle θ to the incident photon in the xy-plane.

3.1. What are the energy and momentum vectors of the incoming and outgoing photons?

3.2. What is the energy and momentum of the particle before the collision?

3.3. What is the energy and momentum of the particle after the collision?

3.4. Use the "Pythagorean relation", along with $\cos^2\theta + \sin^2\theta = 1$, to show that

$$m = \frac{1-\cos\theta}{c^2}\frac{EE'}{E-E'}. \tag{2}$$

Hence, by measuring E, E' and θ, we can detect the mass m of an unknown object. This idea, known as Compton scattering, is used routinely (with X-rays) to probe materials in solid-state physics or biology.

15 **Problem 4:** Suppose an electron (e^-) and a positron (e^+) annihilate each other. As a result of the collision, photons (γ) are produced;

$$e^- + e^+ \to n \times \gamma \tag{3}$$

Show that the number n of photons produced must be $n \geq 2$: namely, explain why $n = 1$ is not allowed.[1]

Problem 5: Two runners A and B start at $x = 0$. At time $t = 0$, and as measured by an observer who stays at rest, they begin to run apart from each other along the x-axis at velocities $-c\beta$ and $+c\beta$ respectively. When the runners measure themselves separated by a distance D, they turn around and run back towards $x = 0$ at the equal and opposite velocity. (This happens at the same time t_0 in frame S.)

Let A' and B' denote the events at which runners A and B turn around, respectively. Let frame S denote the rest frame and S' denote a frame moving at velocity $-c\beta$ relative to S.

10 **5A:** Let us first try to study this problem qualitatively.

- 5A.1. *Sketch* the worldlines of runners A and B, focusing on their trips away from and back to $x = 0$. You will not know all the coordinates of interest, but mark events A' and B', and explain the slopes of any lines which you draw.
- 5A.2. *Sketch* what these worldlines will look like in frame S'. Focus on the relative time ordering of A' and B' and the slopes of lines.

15 **5B:** Now, let us try to quantify what we found before.

- 5B.1. At what time t'_0 in frame S' does event A' occur? Express t'_0 in terms of c, β and/or t_0.
- 5B.2. At $t' = t'_0$ in frame S', how much time has elapsed on runner B's clock? You can express your answer in terms of either t'_0 or t_0.
- 5B.3. Deduce the time t_0 at which the runners turn around in frame S, in terms of D, c and/or β.
- 5B.4. How far apart are the runners at time $t = t_0$ in frame S?

15 **Problem 6 (Tachyons):** A tachyon is a hypothetical particle which has the property that (in one space and one time dimension), for a "tachyon mass" $n > 0$,

$$(nc^2)^2 = (pc)^2 - E^2 > 0. \tag{4}$$

- 6.1. Argue that in any pair of inertial reference frames (moving at speeds not more than c), moving in the x-direction relative to each other, a tachyon is seen in both frames to either move left ($p < 0$) or right ($p > 0$) – however, one frame might see $E > 0$ while the other sees $E < 0$.
- 6.2. Suppose that we have an ordinary particle of mass m. Argue that if the tachyons of "tachyon mass" n existed, then the ordinary particle could spontaneously emit tachyons and generate unboundedly large energy. This instability of ordinary matter is one reason why tachyons are not believed to exist in our actual universe.

[1] *Hint:* Think about the collision in a clever choice of reference frame: something should go wrong when $n = 1$.