PHYS 2170: General Physics 3 for Majors **Fall 2021**

Practice Exam 2

- This exam is open book and open notes. You are not allowed to use any electronic devices besides a calculator, nor allowed to communicate at all about the questions on the exam with any other students for 24 hours after the exam ends.
- You have 2 hours to complete this exam (unless Disability Services has authorized a request for extended time: 50% extra time is 3 hours, 100% extra time is 4 hours). Good luck!

20 **Problem 1:** Consider a hollow metal pipe of length L placed in the air.

1.1. Sound waves in the pipe will be preferably excited at specific wavelengths λ. What are the longest 3 of these wavelengths? Express your answers in terms of L and/or v (the speed of sound in air).

1.2. If the speed of sound is doubled, does the answer to the above question change? Why or why not?

20 **Problem 2:** Consider the following differential equation:

$$\frac{\partial \phi}{\partial t} + c\frac{\partial \phi}{\partial x} + b\frac{\partial^3 \phi}{\partial x^3} = 0. \tag{1}$$

2.1. Plug in the ansatz $\phi = \mathrm{e}^{\mathrm{i}(kx-\omega t)}$, and find a relationship between k and ω.

2.2. What is the phase velocity of a plane wave of wavelength λ?

2.3. What is the group velocity of a plane wave of wavelength λ?

15 **Problem 3 (Perfect destructive interference?):** Consider a string on the half-line $x \geq 0$, on which waves propagate at speed v, according to the linear wave equation. Suppose that an incoming wave packet of arbitrary shape is sent in from large x towards two observers, located at $x = a$ and $x = 2a$ on the line. Let $y(x, t = 0) = h(x)$, and assume that $h(x) = 0$ for $x \leq 5a$.

3.1. At very early times, do we expect $y(x, t) = h(x - vt)$ or $y(x, t) = h(x + vt)$? Why?

3.2. Now suppose that the right observer at $x = 2a$ organizes the shaking of the boundary of the string at $x = 0$, such that a right moving wave is sent from $x = 0$ to perfectly destructively interfere with the incoming wave, such that $y(2a, t) = 0$ for all time t. Show that this is possible to arrange, and describe how it could be done: namely, what is the appropriate solution to the wave equation?

3.3. Is it possible for the shaking to be done in such a way that both $y(2a, t) = y(a, t) = 0$ as well for all times t? Why or why not?

Problem 4 (Laser): A laser generates highly coherent light at almost a perfectly fixed frequency f_0. It achieves this using quantum mechanical transitions between energy levels in atoms (we'll discuss this soon!), which we assume emit light at exactly frequency f_0 in the atom's rest frame.

However, the atoms are in fact in motion in a gas, due to thermal energy. Suppose that the typical speed of atoms making up our laser is about $v \approx 300$ m/s, and atoms are equally likely to be found moving to the left or right. Assume that the atoms live in a one-dimensional cavity of length L.

10 **4A:** Due to the relative motion of the atoms, what is the maximal frequency that we might observe an atom emit? What is the minimal frequency? Denote with Δf this spreading in frequency, and show

$$\Delta f = \frac{2v}{c} f_0. \tag{2}$$

You should use that $v \ll c$ in order to simplify your answer (via Taylor expansion).

15 **4B:** For the device to properly operate as a laser, we would like for the device length L to be chosen such that electromagnetic waves are resonant (i.e. form natural standing waves) at a frequency f which is within the range found above. Assume that the laser operates with "fixed" boundary conditions on the electromagnetic field on each end.

4B.1. Find the resonant frequencies of the cavity. Express your answer in terms of c, L, v and/or f_0.

4B.2. Suppose the laser resonates at frequency f_n (the n^{th} excitable frequency of the cavity). What criterion needs to be satisfied if no other normal modes of the cavity are to be excited?

4B.3. If $f_0 = 5 \times 10^{14}$ Hz, find the constraint on the length L of the cavity so that your condition above is satisfied.

15 **Problem 5:** Suppose we are trying to paint a reflective coating on a device, made out of a material whose index of refraction is 2. Paint A has index of refraction 1.5, while paint B has index of refraction 2.5. If we want to strongly reflect a given wavelength of light λ, what is the thinnest layer of paint we could put over the device? Express your answer in terms of λ and any needed constants. As part of your answer, you need to deduce whether it is better to use paint A or B! Assume the air around the device has index of refraction 1.

15 **Problem 6:** Suppose I take a string of length L and mass per unit length μ, held at fixed tension T. At the left end ($x = 0$), I tie the string down ($y = 0$); at the right end, I manually shake the string:

$$y(L, t) = A \cos(\Omega t), \tag{3}$$

where A is a constant (the amplitude of the shaking) and Ω is the angular frequency with which I shake the string.

6.1. Solve the wave equation, looking for a solution $y(x, t)$ which also oscillates at angular frequency Ω.

6.2. You should find that the response is extreme for certain values of Ω. Why does this happen?

15 **Problem 7:** Three loudspeakers emitting sound waves of wavelength λ are placed at the corners of an equilateral triangle. Suppose that each loudspeaker is in phase with the other two.

7.1. Explain why there will be constructive interference at the center of the equilateral triangle.

7.2. Assuming that the side length L of the equilateral triangle is very large ($L \gg \lambda$), determine (approximately) where are the points closest to the center of the triangle at which there is perfect destructive interference of sound. Quantitatively state how far each point is from the center.