PHYS 2170: General Physics 3 for Majors **Fall 2021**

Practice Exam 3

- This exam is open book and open notes. You are not allowed to use any electronic devices besides a calculator, nor allowed to communicate at all about the questions on the exam with any other students for 24 hours after the exam ends.
- You have 2 hours to complete this exam (unless Disability Services has authorized a request for extended time: 50% extra time is 3 hours, 100% extra time is 4 hours). Good luck!

20 **Problem 1:** Consider an electron in the hydrogen atom.

1.1. If the z-component of angular momentum is $3\hbar$, what is the lowest possible energy of the state?

1.2. How many possible states are consistent with $n = 6$ and $l = 1$?

1.3. How many possible states are consistent with $n = 6$ and $m = 1$?

20 **Problem 2:** Consider a particle of mass m, in Earth's gravitational field (acceleration due to gravity is denoted by g). Due to quantum mechanical effects, the particle will not sit exactly at rest on the ground, but will actually hover very slightly above the ground due to quantum fluctuations.

2.1. Use the Heisenberg uncertainty principle to estimate the typical distance d of the particle above the ground.

2.2. Estimate d for an electron (mass $m \approx 10^{-30}$ kg); comment on the result.

Problem 3 (Radioactivity): Let us consider a toy model for how a radioactive (unstable) nucleus decays. Consider an electron of mass m and energy $U_0 < E < U_1$ in the potential

$$U(x) = \begin{cases} \infty & x < 0 \\ U_0 & 0 < x < a \\ U_1 & a < x < b \\ 0 & x > b \end{cases} . \tag{1}$$

10 **3A:** Let us first describe the classical motion of the electron inside the region $0 < x < a$.

3A.1. Explain why classically, the electron will stay in the region $0 < x < a$ for all time.

3A.2. What is the classical velocity of the electron in this box, in terms of E, U_0, m and/or $\hbar$?

3A.3. Calculate the time τ that it takes for the electron to travel from $x = a$ to $x = 0$.

15 **3B:** Now let us take into account quantum tunneling.

3B.1. Estimate the probability P that, if the electron hits $x = a$, it tunnels to the other side.

3B.2. Deduce that the typical time scale over which the electron can escape the nucleus (i.e., the nucleus undergoes some radioactive decay) is $\tau_{\text{decay}} = \tau/P$, and express τ_{decay} in terms of m, E, U_0, U_1, $\hbar$ and/or τ.[1]

[1] *Hint:* Estimate the number of times we must hit the wall to have, e.g., a 50% chance to tunnel out.

Problem 4: A quantum mechanical particle is trapped inside a one dimensional box $0 \le x \le L$, and has the wave function

$$\Psi(x) = \begin{cases} A\sqrt{x(L-x)} & 0 \le x \le L \\ 0 & \text{otherwise} \end{cases} . \tag{2}$$

15 **4A:** Let us first deduce some basic properties of this wave function.

4A.1. By demanding the overall wave function is normalized, calculate the constant A. You can assume A is a positive real number in your answer.

4A.2. What is the probability that, upon measuring the position of the particle, we would find it to have a position in the left quarter of the box? Namely, evaluate $\mathbb{P}(0 \le x \le \frac{L}{4})$.

5 **4B:** Does this particle have a well-defined energy? Explain your answer.

Problem 5 (Pigments): A typical pigment molecule has the structure sketched in Figure 1. The bonds highlighted in red are special: they are called conjugated π bonds. Suppose there are N conjugated π bonds: then each bond contributes one "free" spin-1/2 electron, of mass m, which may move up and down the chain of bonds freely.

15 **5A:** Suppose N is an even number. If each bond has length a, when N is large, we may thus approximate these electrons as moving in a particle in a box of width $L = a(N-1) \approx Na$. Using the Pauli exclusion principle and the energy levels of a single quantum mechanical particle for the particle in a box, describe which energy levels in the box are filled and which are empty. Remember to account for the spin of the electron!

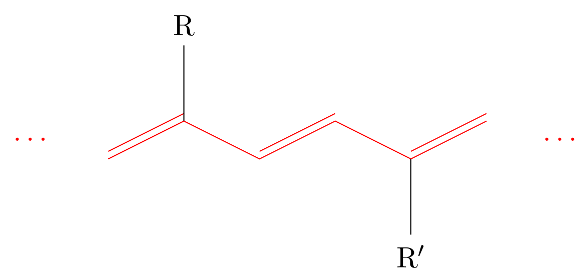

Figure 1: A pigment molecule consists of a chain of conjugated π bonds.

10 **5B:** Now, suppose we send a photon of wavelength λ at the pigment molecule.

5B.1. What is the largest value of λ such that the photon can be absorbed by an electron in the pigment molecule? When the photon is absorbed, an electron must be able to jump to an unoccupied state in the box. You should find that $\lambda \approx KN$ in the limit when $N \gg 1$ – what is the value of K?

5B.2. Evaluate numerically the value of K, given that $m \approx 9 \times 10^{-31}$ kg and $a \approx 10^{-10}$ m.

5B.3. A typical pigment molecule might have a chain with $N \approx 15$. Does λ correspond to a photon in the visible spectrum?

5B.4. Suppose I give you 2 pigment molecules, one of which is red, and one of which is blue. Which pigment molecule do we expect has a longer chain of conjugated π bonds? Note that if a pigment molecule effectively absorbs light at a given wavelength λ_0, light at wavelength λ_0 *can't reflect* easily off the molecule, and so we will see *less light* reflected back at us at λ_0.

Problem 6: Consider a quantum mechanical particle of mass m, attracted to another object via the potential

$$U(r) = -\frac{C}{r^\alpha}, \tag{3}$$

where $\alpha, C > 0$ are constants. Here r denotes the distance from the mass m to the object (which we assume is stationary).

6A: Let's begin by solving the equations of motion classically for the particle's motion. Assume it is traveling in a circular orbit.

6A.1. What is the magnitude of the force F that is felt by the orbiting particle?

6A.2. Relate F to centripetal acceleration to deduce the velocity v of a particle if it is in a circular orbit of radius r.

6B: Now let us try to look for the quantum mechanical bound states of this system using the Bohr quantization condition.

6B.1. Show that there is a critical value of α (call it α_*), such that the nature of the solutions for $\alpha < \alpha_*$ and $\alpha > \alpha_*$ differ in a qualitatively important way.

6B.2. Do you think the results of Bohr quantization are correct for all α? Why or why not?