

Physics 4410
Quantum Mechanics 2

Lecture 14

Metals and insulators

September 25, 2020

Activity 1: Solid with two bands

Consider an electron hopping in a one dimensional metal with

$$H = \sum_{n=-\infty}^{\infty} [(\alpha + \gamma(-1)^n) |n\rangle\langle n| - \beta|n\rangle\langle n + 1| - \beta|n + 1\rangle\langle n|].$$

This is a toy model for an electron moving in an atomic chain of alternating atoms: XYXY... . Why?

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(a) Show that $[H, T_2] = 0$.

- (b) Use Bloch's Theorem to reduce the problem of diagonalizing H to diagonalizing a 2×2 matrix.

- (c) Find the eigenvalues of H . Plot them in an appropriate Brillouin zone.

1. Describe how a Fermi gas of electrons occupies the bands of a real solid.

2. Describe when we have a metal, and when we have an insulator.

Activity 2: Criterion for band insulators.

Suppose I have a lattice model with $[H, T_2] = 0$, and $1/3$ of the allowed electronic states are occupied. Is this a metal or an insulator?