Physics 4410
Quantum Mechanics 2

Lecture 17

The hyperfine interaction

October 7, 2020

Activity: The hyperfine interaction.

Explain why there is a small interaction between the spin-$\frac{1}{2}$s of the proton and electron in hydrogen:

$$H = \frac{A}{\hbar^2}\mathbf{S}\cdot\mathbf{I}.$$

(a) Describe the Hilbert space of the coupled electron/proton spins.

(b) Write down the matrices S_x, S_y, S_z, I_x, I_y, I_z.

(c) Write down H as a 4×4 matrix. What are its eigenvalues?

(d) Define $\mathbf{F} = \mathbf{S} + \mathbf{I}$. Show that $\mathbf{F}$ obeys the angular momentum algebra.

(e) What is $\mathbf{F}^2$? (Relate it to H.)

(f) Explain the triple degeneracy of the hyperfine Hamiltonian.

(g) Given that $A = 9 \times 10^{-25}$ J in hydrogen, find the wavelength of light that is absorbed by the hyperfine transitions in the ground state of hydrogen.