Physics 4410
Quantum Mechanics 2

Lecture 18

Addition of angular momentum

October 9, 2020

1. Consider angular momentum operators $\mathbf{J}_1$ and $\mathbf{J}_2$ associated with two different spins. Define $\mathbf{J} = \mathbf{J}_1 + \mathbf{J}_2$.

2. Explain

$$\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1.$$

3. Explain

$$j_1 \otimes j_2 = |j_1 - j_2| \oplus (|j_1 - j_2| + 1) \oplus \cdots \oplus (j_1 + j_2)$$

4. A heuristic argument for $1 \otimes 1 = 0 \oplus 1 \oplus 2$ comes from considering vector "multiplication".

Activity 1: Spin-orbit coupling.

In many atoms (and solids) the Hamiltonian for an electron contains the following coupling between orbital ($\mathbf{L}$) and spin ($\mathbf{S}$) angular momentum:

$$H = \mathbf{L} \cdot \mathbf{S}.$$

(a) Suppose that the electron has total orbital angular momentum l. Describe the Hilbert space $|ll_z, ss_z\rangle$ in the "coupled basis".

(b) Describe the spectrum of H along with degeneracies.

Activity 2: Adding three angular momenta.

Suppose we have three spin 1 particles. Describe the total Hilbert space in the uncoupled basis, and in the coupled basis: how many effective spin js are there (for each j)?