Physics 4410
Quantum Mechanics 2

Lecture 19

Clebsch-Gordan coefficients

October 12, 2020

1. Review the addition of angular momenta.

2. What are the Clebsch-Gordan coefficients? Explain how to go from uncoupled to coupled bases, or vice versa.

Activity: Decay of unstable particles.

In particle physics, there is an "isospin" (see Homework 7) which is added via angular momentum addition. Particles with net isospin $\frac{3}{2}$ are called Δs. We can distinguish between particles of different z-isospin J_z by their electric charge.

(a) How many Δ particles do we expect? (What is the dimension of the Hilbert space at $j = \frac{3}{2}$?)

(b) Δs are unstable and will decay into isospin $j_1 = \frac{1}{2}$ and $j_2 = 1$ particles. If we take the Hilbert space of these two species of particles, $\frac{1}{2} \otimes 1$, what do we get?

(c) We want to find the Clebsch-Gordan coefficients. Let's start with something easy – how can we write the coupled state $|j = \frac{3}{2}, m = -\frac{3}{2}\rangle$ in the uncoupled basis?

(d) Apply the raising operator to generate $|j = \frac{3}{2}, m\rangle$.

(e) Use orthogonality to find the states $|j = \frac{1}{2}, m\rangle$.

(f) A crude model for Δ ($|j = \frac{3}{2}, m\rangle$) decay is that we simultaneously measure J_{1z} and J_{2z}. Predict the relative decay rates for all possible Δ particle decays. These results have been confirmed beautifully in experiment.