Physics 4410
Quantum Mechanics 2

Lecture 20

The variational principle

October 14, 2020

1. Let Hamiltonian H have ground state E_0. Show that for any $|\psi\rangle$, $\langle\psi|H|\psi\rangle \geq E_0$. (Assume $\langle\psi|\psi\rangle = 1$.)

Activity 1: Consider the Hamiltonian

$$H = \underbrace{\frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2}_{H_0} + \underbrace{u\delta(x)}_{V}$$

(a) Let $|0\rangle$ denote the ground state of H_0. Evaluate $\langle 0|H|0\rangle$.

(b) Let $|1\rangle$ denote the first excited state of H_0. Evaluate $\langle 1|H|1\rangle$.

(c) Use the variational principle to bound the ground state energy of H as a function of the parameter u, and comment.

2. Consider a particle in one dimension. Show that

$$\langle\psi|H|\psi\rangle = \int \mathrm{d}x \left[\frac{\hbar^2}{2m}\left|\frac{\partial\psi}{\partial x}\right|^2 + V(x)|\psi(x)|^2\right].$$

Activity 2: Consider a harmonic oscillator. Let

$$\psi_{\text{trial}}(x;\alpha) = \left(\frac{\alpha}{\pi}\right)^{1/4} e^{-\alpha x^2/2}.$$

(a) Evaluate $\langle H \rangle$ in the state ψ_{trial}.

(b) Minimize over α to find an upper bound on E_0.