Physics 4410
Quantum Mechanics 2

Lecture 21

The helium atom

October 16, 2020

1. Non-dimensionalize the hydrogen atom Hamiltonian.

2. What is the ground state wave function of hydrogen?

Activity: Helium.

Describe the Hamiltonian of the electrons in the helium atom.

(a) Artificially turn off the electron-electron interaction. What is the ground state energy and ground state wave function?

(b) Now apply the variational principle with the wave function

$$\psi_{\text{trial}}(\alpha) = \frac{\alpha^3}{\pi} \mathrm{e}^{-\alpha(r_1+r_2)} \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}.$$

Determine $\langle H \rangle$ in terms of a few integrals.

(c) Show how to relate

$$I_n = \int_0^\infty \mathrm{d}x\; x^n \mathrm{e}^{-cx}, \quad G(t) = \int_0^\infty \mathrm{d}x\; \mathrm{e}^{(t-c)x}$$

(d) Evaluate $\langle |\mathbf{r}_1|^{-1} \rangle$.

(e) Evaluate $\langle |\mathbf{r}_1 - \mathbf{r}_2|^{-1} \rangle$.

(f) Choose the best value of α.