Physics 4410
Quantum Mechanics 2

Lecture 25

Time-independent perturbation theory: non-degenerate, first order

October 26, 2020

1. Review the set-up of perturbation theory.

2. Find the first order correction to the eigenstates.

Activity 1: Consider $H = H_0 + \lambda V$, with

$$H_0 = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2, \qquad V = \frac{1}{2}m\omega^2 x^2.$$

(a) Use first order perturbation theory to find the energy levels to order λ.

(b) Find the first order correction to the ground state wave function.

(c) Find the exact answer for the energy levels. Compare to perturbation theory.

Activity 2: Fragility of quantum states. Consider a quantum information processor, which is storing a quantum state in the ground state of the following Hamiltonian:

$$H_0 = \sum_{i=1}^{N} a_i S_{z,i}$$

Here $S_{z,i}$ denote spin-z matrices for N spins $i = 1, \ldots, N$. Assume that each $a_i > 0$ is distinct.

(a) What are the eigenstates and eigenvalues of H_0?

(b) Now consider the perturbed Hamiltonian

$$H = H_0 + \lambda \sum_{i=1}^{N} S_x.$$

Find the eigenstates of H to first order in λ.

(c) Calculate the length of the order λ correction to each eigenstate. What happens as N becomes large?