Physics 4410
Quantum Mechanics 2

Lecture 27

Time-independent perturbation theory: degenerate

October 30, 2020

1. Review the set-up and motivation for perturbation theory.

Activity 1: Consider the following Hamiltonian:

$$H = \begin{pmatrix} 1 & \epsilon & 0 \\ \epsilon & 1 & \epsilon a \\ 0 & \epsilon a & 2 \end{pmatrix}.$$

(a) Set $a = 0$. Calculate the eigenvalues/eigenvectors of H exactly.

Consider the following Hamiltonian:

$$H = \begin{pmatrix} 1 & \epsilon & 0 \\ \epsilon & 1 & \epsilon a \\ 0 & \epsilon a & 2 \end{pmatrix}.$$

(b) Now assume $a \neq 0$. Write H in the "$a = 0$ eigenbasis".

Consider the following Hamiltonian:

$$H = \begin{pmatrix} 1 & \epsilon & 0 \\ \epsilon & 1 & \epsilon a \\ 0 & \epsilon a & 2 \end{pmatrix}.$$

(c) Find all eigenvalues of H to second order in ϵ.

2. Describe how to do degenerate perturbation theory.

Activity 2: Stark effect. Consider a hydrogen atom in a perturbing electric field.

(a) Write down the Hamiltonian and the perturbation Hamiltonian.

(b) The $n = 2$ state wave functions are:

$$\psi_{2\mathrm{s}}(r), \quad \psi_{2\mathrm{p}}(r)\cos\theta, \quad \psi_{2\mathrm{p}}(r)\sin\theta\mathrm{e}^{\mathrm{i}\phi}, \quad \psi_{2\mathrm{p}}(r)\sin\theta\mathrm{e}^{-\mathrm{i}\phi}$$

Which of the matrix elements in the $n = 2$ subspace don't vanish?

(c) Qualitatively describe how the electric field splits the degeneracy of the $n = 2$ subspace.