Physics 4410
Quantum Mechanics 2

Lecture 28

Fine structure of hydrogen

November 2, 2020

1. Review the hydrogen atom spectrum.

2. What is the Feynman-Hellmann Theorem?

Activity 1: Relativistic corrections.

(a) What is the leading non-trivial correction to kinetic energy in $1/c$?

(b) Re-write p^2 in terms of H and r^{-1} to evaluate $\langle H' \rangle$.

(c) Evaluate $\langle r^{-1} \rangle$ using the Feynman-Hellmann Theorem.

(d) Using

$$\left\langle \frac{1}{r^2} \right\rangle = \frac{1}{a_0^2 n^3 (l + \frac{1}{2})},$$

describe the perturbation to the hydrogen spectrum.

Activity 2: Spin-orbit coupling. There is also the following correction to the hydrogen atom spectrum:

$$H'_{\mathrm{SO}} = \frac{e^2}{8\pi\epsilon_0 m_{\mathrm{e}}^2 c^2 r^3}\mathbf{L}\cdot\mathbf{S}.$$

(a) Re-write $\mathbf{L}\cdot\mathbf{S}$ in terms of the total angular momentum, $\mathbf{J} = \mathbf{L} + \mathbf{S}$.

(b) Using

$$\left\langle \frac{1}{r^3} \right\rangle = \frac{1}{a_0^3 n^3 l(l+\frac{1}{2})(l+1)},$$

evaluate the perturbations to each energy level.