Physics 4410
Quantum Mechanics 2

Lecture 3

Harmonic oscillator: Introduction

August 28, 2020

1. Describe the classical harmonic oscillator.

Activity: Interatomic forces.

The potential energy between a pair of neutral atoms is modeled as

$$V(x) \approx V_0 \left[\left(\frac{a}{x} \right)^{12} - \left(\frac{a}{x} \right)^{6} \right].$$

(a) Carefully sketch $V(x)$.

(b) What is the location of the minimum of $V(x)$? Call it x_0.

(c) Taylor expand $V(x)$ to order $(x - x_0)^2$.

(d) We can approximate the quantum physics of a pair of atoms of mass m with

$$H = \frac{p^2}{m} + V(x).$$

Using the Heisenberg uncertainty principle, and the Taylor expansion of $V(x)$, estimate the spatial spread of the wave function Δx in the ground state.

(e) Estimate the ground state energy.

(f) When does the harmonic oscillator approximation break down?

(g) Helium does not form a solid, even at zero temperature. For a collection of helium atoms, $m \sim 7 \times 10^{-27}$ kg, $a \sim 10^{-10}$ m and $V_0 \sim 10^{-23}$ J. Can you heuristically explain this observation?