

Physics 4410
Quantum Mechanics 2

Lecture 4

Harmonic oscillator: algebraic solution

August 31, 2020

1. The Hamiltonian of the harmonic oscillator is

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2.$$

What are the units of H , x , p , \hbar , m and ω ?

2. Define the *dimensionless* operators \tilde{H} , \tilde{x} and \tilde{p} .

3. Drop tildes for convenience. Define

$$a = \frac{x + ip}{\sqrt{2}}, \quad a^\dagger = \frac{x - ip}{\sqrt{2}}.$$

Express H in terms of a and a^\dagger .

4. Evaluate commutators between a , a^\dagger and H .

5. a is the lowering operator and a^\dagger is the raising operator:

6. What are the eigenvalues of H ?

7. What is $a|n\rangle$? What is $a^\dagger|n\rangle$?