Physics 4410 Quantum Mechanics 2

Lecture 5

Harmonic oscillator: wave functions

September 2, 2020

1. Review the quantum harmonic oscillator and its solution.

Activity 1: A harmonic oscillator is in the initial state

$$|\psi(0)\rangle = \frac{1}{\sqrt{3}}|2\rangle + \sqrt{\frac{2}{3}}|3\rangle.$$

(a) What is $|\psi(t)\rangle$?

(b) What is $\langle \psi(t) | x | \psi(t) \rangle$?

2. Determine the ground state wave function $\psi_0(x) = \langle x | 0 \rangle$.

3. In general, $\psi_n(x) = \langle x | n \rangle$ can be written as

$$\psi_n(x) = \frac{1}{\sqrt{2^n n!} \pi^{1/4}} \mathbf{H}_n(x) \mathbf{e}^{-x^2/2}$$

Activity 2: Molecular spectroscopy.

The harmonic oscillator can be a good toy model for a chemical bond.

(a) What wavelength(s) of light might an oscillator of frequency ω absorb?

(b) The molecule HCl absorbs light at the following infrared wavelengths. Estimate the frequency ω of the chemical bond. Is there any disagreement between the data and our toy model? Note that c ≈ 3 × 10⁸ m/s and ħ ≈ 10⁻³⁴ J ⋅ s.

λ (nm)
746
915
1198
1764
3465