Physics 4410
Quantum Mechanics 2

Lecture 7

Coupled harmonic oscillators

September 9, 2020

Activity: Covalent bonds. (Teaser) Consider the low energy fluctuations of a diatomic molecule...

1. Review how to solve *separable* differential equations.

2. What are the eigenvalues/eigenfunctions of N **decoupled** oscillators:

$$H = \sum_{i=1}^{N} H_i, \qquad H_i = -\frac{\hbar^2}{2m_i}\frac{\partial^2}{\partial x_i^2} + \frac{m_i\omega_i^2}{2}x_i^2.$$

3. Consider a harmonic oscillator with

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m} + \sum_{i,j=1}^{N} \frac{1}{2} K_{ij} x_i x_j .$$

Explain the notion of normal mode coordinates.

4. Find the eigenvalues of

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m} + \sum_{i,j=1}^{N} \frac{1}{2} K_{ij} x_i x_j .$$

5. Find the eigenvalues of

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m_i} + \sum_{i,j=1}^{N} \frac{1}{2} K_{ij} x_i x_j .$$

Activity: Covalent bonds.

Consider a diatomic molecule, whose low energy dynamics is

$$H = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + \frac{k}{2}(x_1 - x_2)^2.$$

(a) Find the normal modes of the classical oscillator.

(b) Find and explain the quantum energy spectrum.