Physics 4410
Quantum Mechanics 2

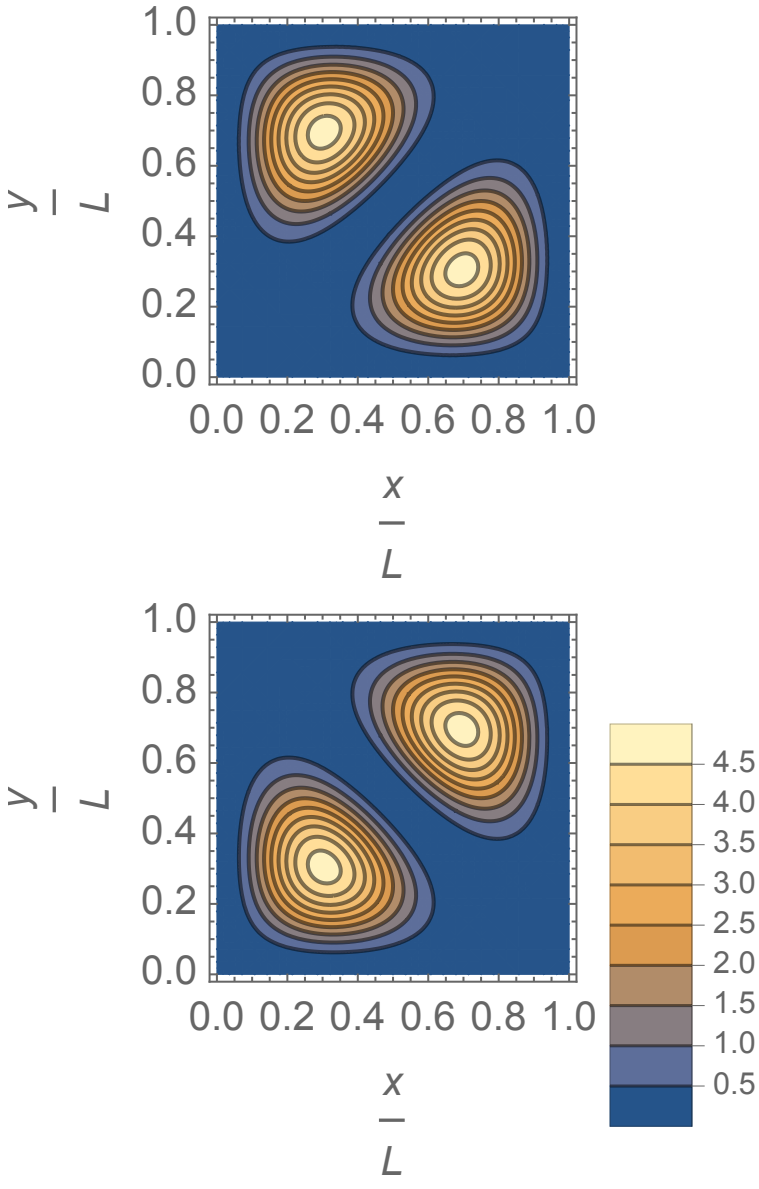
Lecture 9

Exchange interactions

September 14, 2020

1. Review bosonic and fermionic two-particle wave functions.

Activity 1: Two identical particles are in the states ψ_1 and ψ_2 of the 1d infinite square well. Which plot corresponds to bosons/fermions?



2. Expand out $\langle (x_1 - x_2)^2 \rangle \ldots$

3. ...and show that particles are closer together in symmetric vs. antisymmetric states.

4. Explain and justify Hund's Rule.

Activity 2: Magnetic dipoles of diatomic molecules.

In a crude model for a chemical bond, there are three lowest-lying quantum states which an electron can occupy:

$$H = E_1 |\sigma\rangle\langle\sigma| + E_2 \left(|\pi_x\rangle\langle\pi_x| + |\pi_y\rangle\langle\pi_y| \right) .$$

with $E_1 < E_2$.

(a) Consider H_2. Each H contributes one spin-$\frac{1}{2}$ fermion to the chemical bond. Write down the ground state wave function.

(b) Now consider O_2, in which 4 electrons contribute to the chemical bond (roughly speaking). Using the Pauli exclusion principle, suggest which states should be occupied in the ground state?

(c) Electrons are charged and interact via Coulomb forces. With this in mind, argue that the net spin of the electrons in O_2 does not vanish in the ground state.