# PHYS 4410

## Quantum Mechanics 2

## Spring 2023

# Lecture 28

## Symmetry-breaking perturbations

April 3

$\boxed{1}$ $H = \underbrace{H_0}_{\text{solvable}} + \underset{\uparrow\ \text{small}}{V}$.

$H_0$ degenerate?

find basis where $V$ is diagonal in degenerate subspace (use non-deg. PT)

In general, degeneracy in $H_0$ comes from symmetry

Example: $H_0 = A\,\vec{S}\cdot\vec{I}$ (spin-1/2) $\quad V = B(S_x + I_x)$

$\vec{J} = \vec{S} + \vec{I}$

$H = \frac{A}{2}(\vec{J}^2 - \vec{S}^2 - \vec{I}^2) + B J_x$

$[\vec{J}^2, J_x] = 0$

$$H_0 = \left(\begin{array}{ccc|c} \frac{A}{4}\hbar^2 + B\hbar & 0 & 0 & 0 \\ 0 & A/4\hbar^2 & 0 & 0 \\ 0 & 0 & A/4\hbar^2 - B\hbar & 0 \\ \hline 0 & 0 & 0 & -\frac{3}{4}A\hbar^2 \end{array}\right)$$

- Need to diagonalize $V$ w/in deg. subspace.
- Preferred basis chosen by residual rot. sym. $(J_x)$

$\boxed{2}$ $H_0$ : rotation invariant

Hilbert space had 4 dimensions : spin 0 $\oplus$ spin 1

(deg. 1) (deg. 3)

$BJ_x =$ ✓

add $V$: $[J_x, J_y] \neq 0$ $[J_x, J_z] \neq 0$.

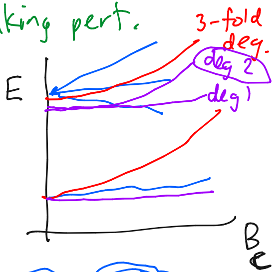
rotation around $x$-axis (generated by $J_x$) is commuting: (Abelian)

no symmetry enforced deg. in $H$.

Lesson: degeneracy lifted by sym-breaking pert.

same $H_0$. $\underline{V = C(\vec{S}\cdot\vec{I})^2}$

rotation invariant.




same $H_0$. $V = D \cdot J_x^2$

$$E = \frac{A}{4}\hbar^2 + D\cdot\hbar^2, \quad \frac{A}{4}\hbar^2, \quad \frac{A}{4}\hbar^2 + D\hbar^2$$

$\boxed{3}$ Consider a 3d isotropic [rot. inv.] oscillator:

$$H_0 = \frac{p_x^2 + p_y^2 + p_z^2}{2m} + \frac{1}{2} m\omega^2 (x^2 + y^2 + z^2)$$

$\vec{p}\cdot\vec{p}$ $\vec{r}\cdot\vec{r}$

$$E^{(0)}_{n_x n_y n_z} = \hbar\omega(n_x + n_y + n_z + 3/2)$$

$[|n_x n_y n_z^{(0)}\rangle]$

$V_1$: break none deg 3

$V_2$: $1+1+1$

$V_3$: $1+2$

First excited: $E = \frac{5}{2}\hbar\omega$

degeneracy 3: $\alpha|100^{(0)}\rangle + \beta|010^{(0)}\rangle + \gamma|001^{(0)}\rangle$

$$\psi(x,y,z) \sim (\alpha x + \beta y + \gamma z)\cdot e^{-\frac{m\omega}{2\hbar} r^2} \qquad (r^2 = x^2 + y^2 + z^2)$$

$$\sim \underbrace{(\alpha' Y_{11} + \beta' Y_{1-1} + \gamma Y_{10})}_{Y(\theta,\phi)} \underbrace{r e^{-r^2}}_{f(r)}$$

3 = these 3 states are $l=1$. symmetry-enforced degen.

$V_1 = \sqrt{x^2+y^2+z^2} = r$, $V_2 = xy$ $V_3 = z^6$ + mirror $[V_3, L_z]$.

$\boxed{4}$ $E = \frac{7}{2}\hbar\omega$, degeneracy of 6:

$|110^{(0)}\rangle$, $|101^{(0)}\rangle$, $|011^{(0)}\rangle$, $|200^{(0)}\rangle$, $|020^{(0)}\rangle$, $|002^{(0)}\rangle$.

In terms of total spin, these states are...

~~$[2j+1 = 6$, then $j = 5/2]$~~

$6 = 1 + 5$
$(0 \oplus 2)$

spin 0: $(a_x^\dagger a_x^\dagger + a_y^\dagger a_y^\dagger + a_z^\dagger a_z^\dagger)|000^{(0)}\rangle$ = spin 0

So: $V \sim r^6$ (rot sym.) ; split $\frac{7}{2}\hbar\omega$ into $\overset{\text{deg.}}{1} + \overset{\text{deg.}}{5}$ levels.

• Sym Group of 3d oscillator has 8 generators [SU(3)]

$\boxed{5}$ Hydrogen atom: $\underbrace{\psi_{n\ell m}}_{\text{rot. sym.}} \longrightarrow E_{n\ell m} = E_n$.

disagrees!

Only rot sym: require $\underbrace{E_{n\ell m} = E_{n\ell}}$

True sym group: $\approx [\text{rotation}]^2$ (6 generators. LRL vector)

consequence of non-rel approx $p^2/2m \sim \underline{\sqrt{(pc)^2+(mc^2)^2} - mc^2}$

$\downarrow$

fine structure

only rot sym.

$$E^{\text{true}}_{n\ell} = E^{\text{NON REL}}_{n} + E^{FS}_{n\ell}$$