PHYS 4410

Quantum Mechanics 2

Spring 2023

Lecture 31

Emission of electromagnetic radiation

April 12

$\boxed{1}$ Today: quantum system interacting w/ EM waves

Dominant interactions:

$$V = -\vec{E}\cdot\vec{p}$$

dipole moment.

$e^{ikr} = e^{i\omega r/c} \to 1$

non-relativistic: $c \to \infty$

$\vec{p} = -e\vec{r}$ (for hydrogen atom)

Light = EM wave: $\vec{E} = E_0 \hat{\varepsilon}\left[e^{i(\vec{k}\cdot\vec{r}-\omega t)} + e^{-i(\vec{k}\cdot\vec{r}-\omega t)}\right]$

polarization (unit vector)

$\omega = c|\vec{k}|$

Via Fermi's Golden Rule:

start in $|i\rangle$, transition rate to $|f\rangle$:

$$R_{i\to f} = \frac{2\pi}{\hbar^2}\delta(\omega_{fi}-\omega)\left|\langle f|E_0\hat{\varepsilon}\cdot\vec{p}\,\cancel{e^{i\vec{k}\cdot\vec{r}}}|i\rangle\right|^2$$

$$\omega_{fi} = \frac{E_f - E_i}{\hbar}$$

"electric dipole approx"

$$= \frac{2\pi}{\hbar^2}E_0^2\,\delta(\omega_{fi}-\omega)\left|\langle f|\hat{\varepsilon}\cdot\vec{p}|i\rangle\right|^2$$

$\boxed{2}$ Incoherent light:

$$E_0 \hat{\varepsilon} e^{i\vec{k}\cdot\vec{r}-i\omega t} \to \sum_{2\hat{\varepsilon}} \int d^3k \; \underbrace{E_0(k)}_{\text{"random phase"}} \hat{\varepsilon}_k e^{i\vec{k}\cdot\vec{r}-i\omega t}$$

$$c_f^{(1)} \sim -\frac{i}{\hbar} \int_0^t ds \; e^{i\omega_{fi}s} \sum_{2\hat{\varepsilon}} \int d^3k \; e^{-ickt} E_0(k) \, \hat{\varepsilon} \cdot \langle f|\vec{p}|i\rangle$$

$$P_{i\to f} = |c_f^{(1)}|^2 \sim \int d^3k \, d^3k_* \underbrace{E_0(k) E_0(k_*)^*}_{\text{average over?}} \cdots$$

$$|E_0(k)| \cdot |E_0(k_*)^*| \cdot e^{i\phi_k - i\phi_{k_*}} \to 0$$

except if $\vec{k} = \vec{k}_*$:

$$P_{i\to f} \sim \int d^3k \; |E_0(k)|^2 \cdots$$

$$R_{i\to f} = \int d^3k \; R_{i\to f}(k)$$

[rates add, not amplit., for incoherent]

3

$$R_{i\to f} = \frac{2\pi}{\hbar^2}\int d^3k\;\delta(\omega_{fi} - c|\vec{k}|)\,\underbrace{|\langle f|\hat{\varepsilon}\cdot\vec{p}\,|i\rangle|^2}\;\underbrace{|E_0(k)|^2}_{\frac{\rho(k)}{2\varepsilon_0}}$$

$\rho(k) \leftarrow$ energy dens.

average over k, $\hat{\varepsilon}$: isotropic average

$$= \frac{1}{3}|\langle f|p_x|i\rangle|^2 + \frac{1}{3}|\langle f|p_y|i\rangle|^2 + \frac{1}{3}|\langle f|p_z|i\rangle|^2$$

$$= \frac{1}{3}|\langle f|\vec{p}\,|i\rangle|^2. \qquad \text{(implicit dot product).}$$

$$\rho(k) = \frac{2}{(2\pi)^3}\,\frac{\hbar\omega}{e^{\hbar\omega/k_BT}-1} \qquad (\omega = c|\vec{k}|)$$

$$\int d^3k\;\delta(\omega_{fi} - c|\vec{k}|)\,\frac{\rho(k)}{2\varepsilon_0} \longrightarrow \int d\omega\,\frac{\hbar\omega^3}{\pi^2 c^3(e^{\hbar\omega/k_BT}-1)}\cdots$$

$$= \int d\omega\;\rho(\omega)$$

$$R_{i\to f} = \frac{\pi}{3\varepsilon_0\hbar^2}\,\rho(\omega_{fi})\,\underbrace{|\langle f|\vec{p}\,|i\rangle|^2}.$$

rate of photons absorbed ($E_f > E_i$)

$E_f < E_i$: stim. emission.

$\boxed{4}$ $H_0 = \frac{p^2}{2m} + \frac{1}{2} m \Omega^2 x^2$ and $p = -ex$

If our initial state $|i\rangle = |3\rangle$, allowed $|f\rangle$?

Write $x = \sqrt{\frac{\hbar}{2m\Omega}}(a + a^\dagger)$.

$\langle f | p | i \rangle = -e \sqrt{\frac{\hbar}{2m\Omega}} \underbrace{\langle f | a + a^\dagger | 3 \rangle}$

$\langle f | \sqrt{3} | 2 \rangle + \langle f | \sqrt{4} | 4 \rangle$

$f = 2$ or $f = 4$.

What is frequency of absorb/emit photon?

$-\Omega = \frac{(5/2 - 7/2)\hbar\Omega}{\hbar} = \frac{E_2 - E_3}{\hbar}$

$\frac{E_4 - E_3}{\hbar} = \Omega$.

Rate: $R_{3\to 2} = \frac{\pi}{3\varepsilon_0 \hbar^2} \rho(\Omega) |\cancel{\langle 2 | a | 3 \rangle}|^2 \left(\frac{e^2 \hbar}{2m\Omega}\right)$

3

Suppose $|i\rangle = |2\rangle$ and $|f\rangle = |3\rangle$...

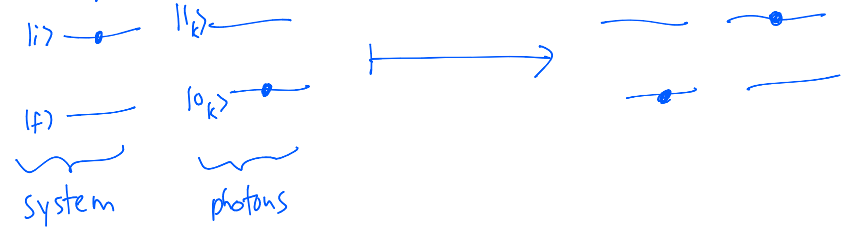
$R_{2\to 3} = R_{3\to 2}$.

runaway heating...

$\boxed{5}$ Einstein: spontaneous emission $|3\rangle \rightarrow |2\rangle$.

McIntyre: stat mech.

Only possible if EM quantize:



Energy:

$E_i + \frac{1}{2}\hbar\omega_k$ $\qquad\qquad$ $E_f + \frac{3}{2}\hbar\omega_k$

V = interaction btwn photon + system

degenerate: $\quad E_f - E_i + \hbar\omega_k = 0$.

$\boxed{6}$ Can generalize time-independent PT ...

$$R^{se}_{i\to f} = \frac{2\pi}{\hbar^2}\int d^3k \, \Big| \underbrace{\langle f| \otimes \langle 1_k| (-\vec{E}\cdot\vec{p}) |i\rangle \otimes |0_k\rangle}_{\left| \underbrace{\langle f|\vec{p}|i\rangle}_{\sim \frac{1}{3}|\langle f|\vec{p}|i\rangle|^2} \cdot \underbrace{\langle 1|\vec{E}|0\rangle}_{\text{QED: } \sqrt{\frac{\hbar\omega}{8\pi^3\varepsilon_0}}} \right|^2} \Big|^2 \delta(\omega_{fi} - ck)$$

(A_{fi})

$$R^{se}_{i\to f} = \frac{2\pi}{\hbar^2}\int \frac{4\pi\omega^2 d\omega}{c^3}\,\delta(\omega_{fi}-\omega)\cdot\frac{1}{3}|\langle f|\vec{p}|i\rangle|^2 \cdot \frac{\hbar\omega}{8\pi^3\varepsilon_0}$$

$$= \frac{\omega_{fi}^3}{3\varepsilon_0\hbar\pi c^3}\,|\langle f|\vec{p}|i\rangle|^2 .$$

rate of <u>spontaneous emission</u>